ESP Math 182 | **Spring 2015**

Worksheet 18

12 March 2015

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1. By the theory of relativity, the length of an object moving relative to an observer changes depending upon how fast the object is travelling (relative to the observer). This phenomenon is called *length contraction / dilation*, or *Lorentz contraction*. By experimental results, the length of the observed object seems to be governed by the equation

$$L(v) = \ell\sqrt{1 - \frac{v^2}{c^2}}.$$

Here, v is the speed of the object in meters per second, ℓ is the length of the object (in meters) at rest (relative to the observer), and c is the speed of light, $c \approx 3 \cdot 10^9$ meters per second.

(a) Find the linear approximation to L at $v = 0$.

(b) Find the quadratic approximation to L at $v = 0$.

(c) Find the cubic approximation to L at $v = 0$.

(d) Using the original formula $L(v)$, what is the error for your answers above?

(e) Using the cubic approximation, estimate:

i. the length of an observed car moving at 100 kilometers per hour, by a pedestrian on the sidewalk, if the car is 3 meters long at rest;

ii. the length of the observed spacecraft *Helios-B*, travelling at $\approx 2.5 \cdot 10^5$ kilometers per hour, by an observer on the earth, if the spacecraft is 32 meters long at rest.

Try the above without a calculator!

(f) Explain what happens to $L(v)$ when the velocity v of the object approaches c. What happens when $v = c$? When $v > c$?

2. Consider the functions

$$f(x) = 2x^2 + 4x - 1 \quad , \quad g(x) = \sin(x) \quad , \quad h(x) = e^{-1/x^2}.$$

(a) Find the linear, quadratic, and cubic approximations for all three functions above around the point $x = 0$.

(b) At $x = 10$, what are the errors for the approximations?

(c) Make an educated guess as to why the errors are as large / small as they are.