ESP Math 182 | **Spring 2015**

Worksheet 17

10 March 2015

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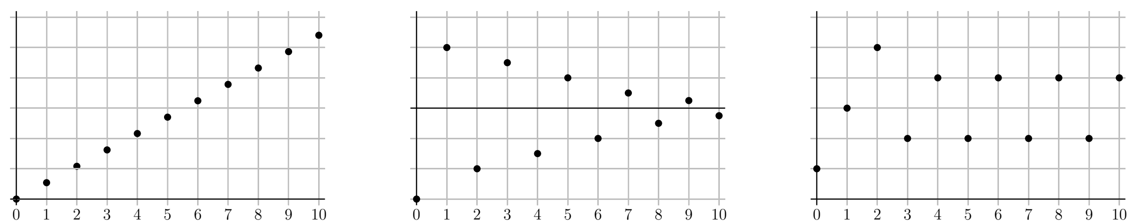
1. **Warm up:** Consider the (first few elements of the) infinite sequences below, assuming that the displayed patterns continue. State which of the following properties are exhibited by each sequence: convergent, divergent, bounded, monotonic.



(a) (b) (c)

2. State whether each sequence $\{a_n\}_{n=0}^{\infty}$ converges or diverges. If it converges, find the limit as $n \to \infty$.

   (a) $a_n = \dfrac{99}{2^n}$

   (b) $b_n = \left(1 - \dfrac{3}{n}\right)^n$

   (c) $c_n = \dfrac{n^3 \sin^3(n)}{n+1}$

   (d) $d_n = \dfrac{2n + 3\sin\left(\frac{\pi n}{4}\right)}{3n+1}$

   (e) $e_n = \dfrac{n^{99}}{e^n}$

3. Consider the sequences $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=2}^{\infty}$, and $\{c_n\}_{n=3}^{\infty}$, where

$$a_n = \frac{2n}{n^2 - 2} \quad , \quad b_n = \cos(\pi n/2 + \pi/4) \quad , \quad c_n = a_n + b_n.$$

(a) Reindex all three sequences so that they begin at $n = 0$.

(b) Which of the sequences converge and which diverge?

(c) What can you say about the sequence $d_n = \frac{c_n}{a_n}$? Does it converge or diverge?

4. Determine if the following statements are true or false. If true, provide some justification. If false, provide a counterexample.

(a) If $\lim_{n\to\infty} [a_n] = 0$ and $\lim_{n\to\infty} [b_n] = \infty$, then $\lim_{n\to\infty} [a_n b_n] = 0$.

(b) If the sequence $\{a_n\}_{n=0}^{\infty}$ converges, then $\{(-1)^n a_n\}_{n=0}^{\infty}$ also converges.