ESP Math 182

Worksheet 16

Spring 2015

5 March 2015

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1. **Warm up:** Answer the following true / false questions. No reasons are necessary.

   (a) The sequence $\{a_n\}_{n=1}^{\infty}$ for $a_n = \frac{1}{n}$ converges.

   (b) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

   (c) If a series $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then a_n converges to 0 (that is, $a_n \to 0$ as $n \to \infty$).

   (d) If a sequence is such that a_n converges to 0, then $\displaystyle\sum_{n=0}^{\infty} a_n$ converges.

2. Use a geometric series to show that

   (a) $0.99999\ldots\ldots = 1$

   (b) $0.5555555\ldots\ldots = 5/9$

   (c) $1.285714285714\ldots\ldots = 9/7$

3. Find an explicit formula for the n-th partial sum $S_n = \displaystyle\sum_{k=0}^{n} a_k$ for each of the following examples, and then evaluate the limit $\displaystyle\lim_{n\to\infty}[S_n]$ to find the exact value of the series, if it converges.

   (a) $\displaystyle\sum_{k=0}^{\infty}(-1)^k$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k+2} - \frac{1}{k+3}$

4. Let a_n, b_n be sequences. Determine, with reasons, which of the following statements are true. Give counterexamples if they are false.

   (a) $\displaystyle\sum_{n=0}^{\infty}(a_n + b_n) = \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n$

   (b) $\displaystyle\sum_{n=0}^{\infty} a_n b_n = \left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right)$