

3 March 2015

1. **Warm up:** Give an example of each of the following objects.

- (a) non-increasing sequence
- (b) increasing sequence
- (c) non-decreasing sequence
- (d) decreasing sequence
- (e) constant sequence
- (f) monotonic sequence
- (g) sequence that is bounded below
- (h) sequence that is bounded above
- (i) bounded sequence

**Bonus:** What are the relations among the objects above? That is, which objects are specific cases of other objects? Write statements using the implication symbol  $\implies$ .

2. Find the limit as  $n \rightarrow \infty$ , or state and prove that the sequence diverges.

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| (a) $a_n = \left(\frac{1}{3}\right)^n$   | (f) $f_n = \frac{3n^2 + n + 2}{2n^2 - 3}$    |
| (b) $b_n = (0.1)^{-1/n}$                 | (g) $g_n = \frac{e^n}{2^n}$                  |
| (c) $c_n = \frac{n}{\sqrt{n^2 + 1}}$     | (h) $h_n = \frac{3 - 4^n}{2 + 7(4^n)}$       |
| (d) $d_n = \frac{n + 2}{\sqrt{n^3 + 1}}$ | (i) $i_n = n \sin\left(\frac{\pi}{n}\right)$ |
| (e) $e_n = \sin(n\pi)$                   |  |

3. Find the limit of  $a_m = \sqrt[m]{m}$  as  $m \rightarrow \infty$ .

4. Find the limit of  $b_k = \left(1 + \frac{1}{k^2}\right)^k$  as  $k \rightarrow \infty$ .

5. Find the limit of  $c_\ell = \sum_{i=1}^{\ell} \frac{1}{i} - \ln(\ell)$  as  $k \rightarrow \infty$ .