ESP Math 182 **Spring 2015**

Worksheet 13

24 February 2015

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1. **Warm Up:** Evaluate the following integrals.

   (a) $\displaystyle\int \frac{dx}{x^2 - 7x + 10}$

   (b) $\displaystyle\int \frac{3x + 6}{x^2(x-1)(x-3)}\, dx$

   (c) $\displaystyle\int \frac{(3x^2 - 2)\, dx}{x - 4}$

   (d) $\displaystyle\int \frac{dx}{x(x^2 + x)}$

   (e) $\displaystyle\int \frac{\sqrt{x}\, dx}{x^3 + 1}$

2. Let $a \neq b$ be constants. Prove the general formula

$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b} \ln\left(\frac{x-a}{x-b}\right) + C.$$

3. Let $f(x) = \frac{1}{1-x^2}$.

   (a) Use partial fractions to evaluate the integral $\int f(x)dx$.

   (b) Using the fact that the derivative of $\tanh^{-1}(x)$ is $1/(1-x^2)$, find another answer to $\int f(x)dx$.

   (c) Knowing that $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ and $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$ and the definition of inverse functions, show that the two answers are equal.

4. Old Man Smith has led a long and productive life working as a painter and raising four beautiful girls. But, there's always been one thing that he wanted to do that he hasn't been able to since he never took Calculus II. His favorite curve is $y = \frac{1}{x}$, he just thinks it looks nice. He was always wondering how much paint a paint can could hold if it were the shape of $y = \frac{1}{x}$ rotated about the x-axis for $x \geqslant 1$.

   (a) Help out Old Man Smith and find the volume of the paint can if its height is H?

   (b) What is the area of the strip between $y = \frac{1}{x}$ and $y = -\frac{1}{x}$ which would run through the diameter of the can?

   (c) Now by letting $H \to \infty$ notice that one of the values from (a) and (b) converges to a finite value and the other becomes infinite. Compute the finite value.

   (d) Hypothetically speaking, if a strip of paper were made that ran through the diameter of the infinitely long paint can, would there be enough paint in the can to cover it?