ESP Math 182 **Spring 2015**

Worksheet 11

17 February 2015

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1. Let $m, n \in \mathbf{Z}$ (the set of integers). Evaluate the following expressions.

   (a) $\displaystyle\int_0^\pi \sin^2(mx)\, dx$

   (b) $\displaystyle\int_0^\pi \sin(mx)\sin(nx)\, dx$

   (c) $\displaystyle\int_0^\pi \cos(mx)\cos(nx)\, dx$

2. Show by differentiation that if $P_n(x)$ is a polynomial of degree n which satisfies the equation $P_n(x) + P_n'(x) = x^n$, then $\int x^n e^x dx = P_n(x)e^x + C$.

3. Let $\Gamma(x) = \displaystyle\int_0^\infty e^{-t}t^{x-1}dt$ for $x > 0$.

   (a) Use integration by parts to show that $\Gamma(x+1) = x\Gamma(x)$ for $x > 0$.

   (b) Show that $\Gamma(1) = 1$.

   (c) Show that $\Gamma(n) = (n-1)!$ for all $n \in \mathbf{N}$ (the set of natural numbers).

4. Find the volume of revolution obtained by revolving the graph of $y = \sin(x)$ between $x = 0$ and $x = \pi$ around the axis $y = 1/2$.