ESP Math 182 **Spring 2015**

Worksheet 10

12 February 2015

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1. A 100-W light bulb has resistance $R = 144$ ohms when attached to household current, where the voltage varies as a function of time in seconds, $V = V_0 \sin(2\pi f t)$, for $V_0 = 110$ volts and $f = 60$ Hz. The power supplied to the bulb is $P = V^2/R$ joules per second and the total energy expended over a time period $[0, T]$ (in seconds) is $U = \int_0^T P(t)dt$ joules. Compute U if the bulb remains on for half an hour.

2. Estimate the value of $\displaystyle\int_0^2 \frac{dx}{x+1}$ using

   (a) the midpoint rule with 2 intervals, and

   (b) the trapezoidal rule with 2 intervals.

3. The hyperbolic cosine function $\cosh(x)$ is defined to be $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$. Find the arc length of the graph of $\cosh(x)$ on the interval $[-\ln(2), \ln(2)]$.

4. Show that if $n \in \mathbf{R}$ and $n \geqslant 2$, then

$$\int \sec^n(t)dt = \frac{1}{n-1}\sec^{n-2}(t)\tan(t) + \frac{n-2}{n-1}\int \sec^{n-2}(t)dt,$$

   and use this to calculate $\int \sec^4(t)dt$. (Hint: note $\sec^n(x) = \sec^{n-2}(x)\sec(x)$)

5. Find $\int (\ln(x))^k dx$ for arbitrary $k \in \mathbf{N}$.

6. For what functions f, g is it the case that $\frac{d}{dx}(f(x)g(x)) = \frac{d}{dx}(f(x))\frac{d}{dx}(g(x))$?