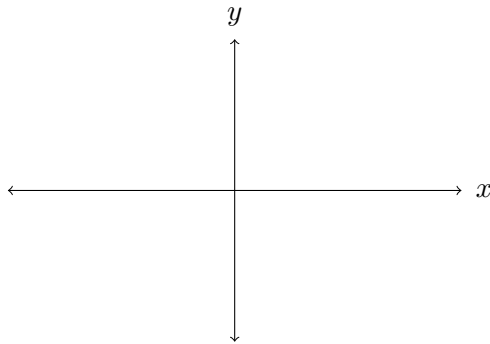


Discussion session 6 - 11 September 2014

1. (a) Draw the function $f(x) = (x - 2)^2 + 1$ on the axes below. *Remember to label the axes with a scale.*



- (b) Using the definition of derivative, find $f'(3)$ and draw the tangent line to f at $x = 3$ on the graph above.
- (c) Find the equation of the tangent line to f at $x = 3$.
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2. Consider the function

$$f(x) = \frac{x}{2x - \sqrt{x^2 - 1}}.$$

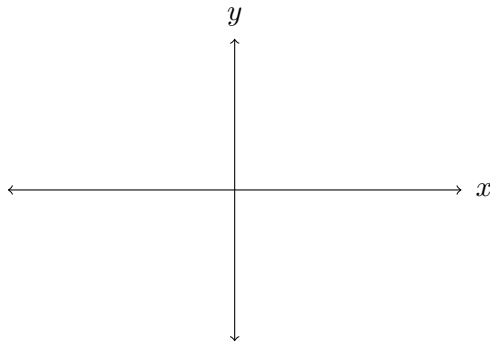
- (a) What is the domain of f ?
- (b) What is the range of f ?
- (c) What are all the horizontal asymptotes of f ?

3. Consider the piecewise function

$$f(x) = \begin{cases} x + 5a & \text{if } x \leq -3, \\ -ax + b & \text{if } -3 < x < 3, \\ 2x + 10b & \text{if } x \geq 3. \end{cases}$$

(a) Find values of a and b such that f is continuous everywhere.

(b) Draw the graph of f on the axes below. *Remember to label the axes with a scale.*



4. (a) Explain in your own words what a *zero of a function* is.

(b) Show that the polynomial $f(x) = x^5 + \pi x^3 - e$ has at least one zero.

5. State whether each question is true or false, with justification.

(a) The intermediate value theorem can be applied to $f(x) = \frac{1}{x^2 - 4x + 5}$ on any interval $[a, b] \subset \mathbb{R}$.

(b) If the limit of a function exists at every point in its domain, then the function is continuous everywhere.

(c) The function $\tan(x)$ is defined for all $x \in \mathbb{R}$.