Calculus I **Review** **Fall 2014**

Discussion session 26 - 20 November 2014

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1. **Topic**: Definite integrals, FTC, substitution

Submitted by:
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   (a) Compute the indefinite integral $\int \cos^3(x) + \tan^2(x)dx$.

   (b) Find the area under the curve $10x + \sin(x)$, from 0 to 2π.

   (c) Find the area between $f(x) = x^3$ and $g(x) = (x-2)^2$ from $x = -1$ to $x = 3$.

   (d) Compute the definite integrals below.

$$\int_0^3 e^{7x} - 2x^2 dx \qquad\qquad \int_{-2}^3 \frac{3x^2 - \sqrt[3]{x^2} + 5}{\sqrt[3]{x}} dx.$$

   (e) Compute the integral $\int_0^1 \sin(e^x)e^x dx$ using substitution.

   (f) If $F(t) = \int_1^{5t} 3x^2 + 4x + 1 dx$, find $F(1)$ and $F'(1)$.

   (g) If $f(x) = \int_{1/x}^{x^2} \frac{1}{t^4+1} dt$, find $f'(x)$.

2. **Topic**: Linear approximation, MVT, l'Hôpital's rule

Submitted by:
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   (a) Approximate $\sqrt{24}$ and $\sqrt[4]{82}$ using linear approximation.

   (b) Find the best linear approximation of $f(x) = xe^x$ at $a = \ln(2)$.

   (c) Evaluate the limits $\lim_{x\to 0}\left[\frac{5^x - 1}{x}\right]$ and $\lim_{x\to 0}\left[\frac{x}{2x}\right]$.

   (d) Show that l'Hôpital's rule is useless for $\lim_{x\to\infty}\left[\frac{x + \cos(x)}{x - \cos(x)}\right]$.

   (e) Find the critical point of $f(x) = x^2$ that passes through the line $y = 2x$ on the interval $[0, 2]$.

   (f) Suppose that we know that $f(x)$ is continuous and differentiable on $[6, 15]$. Let's suppose that we know that $f(6) = -2$ and that we know that $f'(x) \leqslant 10$. What is the largest possible value for $f(15)$?

3. **Topic**: Graphing, optimization, related rates

Submitted by:
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   (a) Sketch the graphs of

$$f(x) = \frac{e^{\cos(x^2)}}{x^2 - 4} \qquad , \qquad g(x) = \frac{x}{x^2 + 1}.$$

(b) A rectangle is to be formed with its lower left corner at the origin and the upper right corner on the curve $y = e^{-2x+1}$. Find the maximum area of such a rectangle.

(c) The surface area of a box with square base and no top lid is 30in^2. What is the largest possibel volume?

(d) Air is being pumped into a spherical balloon at a rate of $\pi\text{cm}^3/\text{min}$. Determine the rate at which the radius of the balloon is increasing when the circumfrence of the balloon is 20cm.

(e) Find the rate of change of the volume of a balloon if its radius is 20 centimeters and it increases at a rate of $.2\text{in}^2/\text{sec}$.

(f) A pole is leaning against a wall. What is the speed at which the upper part is falling if the speed by which the lower part is moving to the right is 3ft/sec. and its distance from the wall is 8ft? The length of the pole is 17ft.

(g) A pole is sliding down a wall angled at 30°. The variable is the height of the ladder at time t, and x is the distance of the ladder's bottom from the wall. Suppose that the top is sliding dawn the wall at a rate of 8ft/sec. Calculate $\frac{dx}{dt}$ when the height of the ladder is 4ft.

4. **Topic**: Extrema, monotonicity, concavity

Submitted by:
Collin M
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Jose A
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Josh J
Alina S.

(a) Sketch the graph of $f(x) = \frac{1}{\sin(\ln(x)^2}$ and find all inflection and critical points.

(b) Find the extrema for the functions $f(x) = x^2 - 3x$ on the interval $[1, 5]$ and $g(x) = x\sin(x)$ on $[0, 10]$, and determine whether they are local or absolute.

(c) Find the intervals of monotonicity of $f(x) = \frac{x^2+1}{x^2-1}$ and $g(x) = x^4 - 4x$.

(d) Find all critical points and inflection points of $f(x) = \frac{(x^2+2)^2}{x+1}$ and $g(x) = \frac{x}{x^2-1}$.

(e) use the second derivative test to find the maximum and minimum of $f(x) = 3x^4 - 4x^3$.

5. **Topic**: Chain rule, implicit differentiation

Submitted by:
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Joseph R
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Fariz K
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(a) Differentiate the following expressions using the chain rule:

$$\sin(5^x) + 3 \qquad (x + \cos(x))^9 \qquad e^{\tan(x)} \qquad \ln(\cos^5(3x^4))$$

(b) Find the derivative of the following functions using implicit differentitation:

$$5x^2y^3+6xy^4 = 7 \qquad \sin^2(y)+3\cos(x) = 6 \qquad 5x+6y = \cos(x^2)y \qquad \cos^5((3y4)^x) = y+x.$$