**Calculus I** # Integrals and word problems **Fall 2014**

**Discussion session 21 - 4 November 2014**

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1. Take the derivative and indefinite integral, with respect to $x$, of the following functions:

   (a) $x$

   (b) $e$

   (c) $x^e$

   (d) $e^x$

   (e) $e^e$

   (f) $e^{x^e}$

2. Here are some useful identities:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

   Use these identities to help simplify the following expressions (that is, find an answer only in terms of $n$).

   (a) $\displaystyle\sum_{k=1}^{n}(2k+3k^2)$

   (b) $\displaystyle\sum_{j=0}^{n-3}\frac{j^2+2j+1}{2j+2}$

   (c) $\displaystyle\sum_{k=2}^{n+2}\left((k-2)^3+k^2\right)+\sum_{k=3}^{n+3}\left((k-3)^3-2k\right)-\sum_{k=0}^{n}2k^3$

3. An 80cm piece of wire is cut into two pieces. One piece is bent into an equilateral triangle. The other piece is bent into a rectangle with one side four times the length of the other side. Determine where, if anywhere, the wire should be cut to maximize the area enclosed by the two figures.

4. A pendulum, attached by a string to the wall at $(0, 0)$ and the ball held at $(8, -6)$, begins to swing downward. How close will the ball come to the point $(9, -9)$? Assume that before being let go, the string is taut, and the ball has radius .1 units.

5. A hockey player hits two hockey pucks one second apart from the same spot. After being hit, the first hockey puck slides at speed $v_1$ meters per second directly ahead of the hockey player, and the second slides at speed $v_2$ meters per second at an angle of $30°$ to directly ahead of her. Assume the ice is frictionless and endless.

   (a) After $t = 1$, at what rate is the distance between the two hockey pucks changing?

   (b) If the hockey player begins skating after the second puck at a constant speed 10 seconds after hitting the first puck, at least how fast must she be skating to reach the second puck?