

28 April 2016

1. Compute the derivative of the following functions, with respect to x .

(a)
$$\frac{\cos(4x^2)^{x-5x^2} + \arcsin(x^{1/2} + 1/2)}{\pi + 2}$$

(b)
$$\int_0^x \sin(e^{t^2}) dt$$

(c)
$$\int_x^{x^2} \ln(r) + \sqrt{r} dr$$

(d)
$$\sum_{k=1}^n x^k$$

2. Water is flowing into a spherical-shaped fishbowl of diameter 30 cm at a rate of $1 + t/2$ cm³/sec, where t is time in seconds. A rubber duck is floating on the water in the bowl directly in the center of the water surface. Assume at time $t = 0$ the bowl is empty and water begins to flow into it at $t = 0$.

(a) Give an expression, as a function of time, for the vertical distance between the center of the rubber duck and the bottom of the fishbowl.

(b) Give an expression, as a function of time, for the horizontal distance between the center of the rubber duck and the sides of the fishbowl.

(c) At what time is the surface area of the water facing the top of the fishbowl the largest?

(d) At what rate is the surface of the water facing the top of the fishbowl changing at time $t = 5$ seconds?

(e) Assuming the bowl is perfectly spherical and the rubber duck has a height of 3 cm above the water, at what time will there not be enough vertical space in the bowl for the duck?

3. Describe over what intervals the function $f(x) = 2x^5/5 - x^4/2 - 4x^3$ is:

(a) increasing

(b) decreasing

(c) concave up

(d) concave down