Worksheet 28

21 April 2016

1. **Warm up:** Fill in the blanks using $<, \leqslant, =, \geqslant, >$.

   (a) For any function f,

   $$\int_a^b |f(x)|dx \quad \underline{\hphantom{xxx}} \quad \left|\int_a^b f(x)\ dx\right|.$$

   (b) For an even function g,

   $$\int_{-a}^a |g(x)|dx \quad \underline{\hphantom{xxx}} \quad \left|\int_{-a}^a g(x)\ dx\right|.$$

   (c) For an odd function h,

   $$\int_{-a}^a |h(x)|dx \quad \underline{\hphantom{xxx}} \quad \left|\int_{-a}^a h(x)\ dx\right|.$$

2. Rewrite the following indefinite and definite integrals with the given substitution and evaluate them.

   (a) $\displaystyle\int 4x + 2\ dx \quad , \quad u = 2x + 1$

   (b) $\displaystyle\int \frac{5}{(x-2)^2}\ dx \quad , \quad u = x - 2$

   (c) $\displaystyle\int 3x^2 + 6x - 1\ dx \quad , \quad u = 3x - 1$

   (d) $\displaystyle\int_{-1}^{3/2} \cos(\pi\theta)\ d\theta \quad , \quad \alpha = \pi\theta$

3. Graph the areas represented by the following integrals and evaluate them *using geometry.*

(a) $\int_{-5}^{8} 3x - 2 \, dx$

(b) $\int_{-1}^{7/2} f(x) \, dx$ where $f(x) = \begin{cases} -1 & x \in (k, k+1] \\ 1 & x \in (k+1, k+2] \end{cases}$ for all even integers k

(c) $\int_{-4}^{4} \sqrt{16 - x^2} \, dx$

4. Evaluate the following integrals using any tools you know.

(a) $\int_{0}^{\pi/2} \sin^4(\theta) \cos(\theta) \, d\theta$

(b) $\int_{0}^{4} \frac{1}{x^2 + 1} \, dx$

5. What is one topic in Math 180 that you

(a) know very well?

(b) do not know at all?