Worksheet 27

19 April 2016

1. **Warm up:** Answer the following questions with True / False.

   (a) The antiderivative and indefinite integral of a function are the same thing.

   (b) An even function cannot be an antiderivative of an odd function.

   (c) The definite integral of an odd function must be 0.

2. Take the derivative and definite integral from C (some constant) to t, with respect to x, of the following functions.

   (a) x (d) e^x

   (b) e (e) e^e

   (c) x^e (f) ee^{ex}

3. Give short answers, with justification, to the following questions.

   (a) Why is the integral of $\sin(x)$ on $[-2\pi, 2\pi]$ equal to 0?

   (b) Why is the integral of $\sin(x + \pi/2)$ on $[-5\pi/2, 3\pi/2]$ equal to 0?

   (c) Why is the integral of $\sin(x) + 1$ on $[-5, 5]$ not equal to 0?

4. For every pair of type of functions below, indicate if the sum and product are even, odd, or have no symmetry. Your answer must be true for *all* examples of function types given.

   (a) two even functions

      i. sum

      ii. product

   (b) two odd functions

      i. sum

      ii. product

   (c) an odd function and an even function

      i. sum

      ii. product

   (d) an even function and a constant function

      i. sum

      ii. product

   (e) an even function and a periodic function

      i. sum

      ii. product

5. What do you think $\int_a^\infty f(x)\, dx$ means? Come up with a reasonable definition and apply it to calculate $\int_0^\infty e^{-x}\, dx$.