

14 April 2016

1. **Warm up:** Recall the general form of a right Riemann sum of a function  $f(x)$  over an interval  $[a, b]$  with  $n$  subintervals.

$$R_n = \sum_{i=1}^n f\left(a + i \cdot \frac{b-a}{n}\right) \cdot \frac{b-a}{n}$$

Using this, for the right Riemann sum below, identify the function, the interval, and the corresponding definite integral once the limit is taken.

$$R_n = \sum_{k=1}^n \frac{5/n}{3 + \frac{5k}{n}} \quad f(x) = \quad [a, b] = \quad \lim_{n \rightarrow \infty} [R_n] = \int$$

2. (a) Given that  $\int_0^{\pi/2} \cos(\theta) - 2\sin(\theta)d\theta = -1$ , compute  $\int_{\pi/2}^0 4\cos(\theta) - 8\sin(\theta)d\theta$ .

- (b) Given that  $\int_0^3 h(x) + 5 dx = 7$  and  $h$  is even, compute  $\int_{-3}^3 2h(x) - 9 dx$ .

3. Let

$$f(x) = \begin{cases} -2 & \text{if } x < 1, \\ 5 - 2x, & \text{if } x \geq 1. \end{cases}$$

Graph  $f(x)$  and compute  $\int_{-2}^4 f(x)dx$  and  $\int_{-2}^4 |f(x)|dx$ .

4. A base jumper dives off a cliff that is 200 meters high.
- (a) Assume the acceleration due to gravity is 9.8 meter per second squared. What is her velocity function  $v(t)$ , where  $t$  is time in seconds?
- (b) What is her height function  $h(t)$ ?
- (c) She pulls her chute when she is 100 meters down. What is her speed in this instant?
5. Find the absolute maximum of the function  $f(x) = \frac{1}{1 + |x|} + \frac{1}{1 + |x - 2|}$ .
6. A triangle with side lengths  $a, b, c$  varies with time  $t$ , but its area never changes. Let  $\theta$  be the angle opposite the side of length  $a$ , and suppose  $\theta \in [0, \pi/2]$  for all time  $t$ .
- (a) Express  $\frac{d\theta}{dt}$  in terms of  $b, c, \theta, \frac{db}{dt}$  and  $\frac{dc}{dt}$ .
- (b) Express  $\frac{da}{dt}$  in terms of the same quantities as above.