ESP Math 179

Worksheet 24

Spring 2016

7 April 2016

---

1. **Warm up:** Recall the general forms of the nth Riemann sum of f over $[a, b]$. Identify each of the following expressions as either the left, right, or midpoint Riemann sums.

   (a) $\displaystyle\sum_{i=1}^{n} \frac{b-a}{n} f\left(a + (i-1) \cdot \frac{b-a}{n}\right)$

   (c) $\displaystyle\sum_{i=1}^{n} \frac{b-a}{n} f\left(a + (i-1) \cdot \frac{b-a}{n} + \frac{b-a}{2n}\right)$

   (b) $\displaystyle\sum_{i=1}^{n} \frac{b-a}{n} f\left(a + i \cdot \frac{b-a}{n}\right)$

   (d) $\displaystyle\sum_{i=0}^{n-1} \frac{b-a}{n} f\left(a + i \cdot \frac{b-a}{n}\right)$

2. (a) Give the nth right Riemann sum of $f(x) = x^2 + x$ over $[0, 5]$. Leave it in summation (sigma) notation.

   (b) Simplify your previous answer, using the following identites:

   $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

   (c) Take the limit of your previous answer, as $n \to \infty$.

   (d) Evaluate the antiderivative of $x^2 + x$ with constant $c = 0$ at $x = 5$ and compare it to your previous answer.

3. Graph $f(x) = 2 - |x|$ and compute the area under its graph from $x = -1$ to $x = 2$.