ESP Math 179 **Spring 2016**

Worksheet 22

31 March 2016

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1. **Warm up:** Evaluate the following limits using l'Hôpital's rule.

   (a) $\lim_{x\to 1}\left[\dfrac{x^n-1}{x-1}\right]$ (b) $\lim_{z\to 0}\left[\dfrac{\tan(4z)}{\tan(7z)}\right]$ (c) $\lim_{x\to 0^+}[x^x]$

2. Suppose that $\lim_{x\to\infty}[f(x)]=0$ and $\lim_{x\to\infty}[g(x)]=\infty$. Show, by rearranging the functions f and g, how $\lim_{x\to\infty}[f(x)g(x)]$ may be viewed as a limit

   (a) indeterminate of the form $\frac{\infty}{\infty}$,

   (b) indeterminate of the form $\frac{0}{0}$.

3. The mean value theorem (MVT) says if f is continuous on $[a,b]$ and differentiable on (a,b), then there is at least one c between a and b such that $\frac{f(b)-f(a)}{b-a}=f'(c)$.

   (a) Find a function f and an interval $[a,b]$ where there is only one such c.

   (b) Find a function f and an interval $[a,b]$ where there are exactly two such c's.

   (c) Find a function f and an interval $[a,b]$ where there are exactly $n\,3$ such c's. In this case your a and b may depend on n.

4. Evaluate the following limits using l'Hôpital's rule (and any other tools you know).

(a) $\lim_{x\to 2^+}\left[\dfrac{1}{x-2}-\dfrac{1}{\ln(x-1)}\right]$

(b) $\lim_{x\to 0^+}\left[(e^x+x)^{\frac{1}{4x}}\right]$

(c) $\lim_{x\to 0^+}\left[\dfrac{\sin(x)-x}{x^3}\right]$

(d) $\lim_{x\to 0^+}\left[(\cos(x)-1)^x\right]$

(e) $\lim_{x\to \pi/2^+}\left[\dfrac{\sec(x)}{1+\tan(x)}\right]$

(f) $\lim_{x\to 0^+}\left[\dfrac{2\ln(e^x-1)}{\ln(3x)}\right]$