ESP Math 179 **Spring 2016**

Worksheet 21

29 March 2016

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1. **Warm up:** Fill in the blanks for the statement of the mean value theorem (MVT):

   Let f *be* _____ *on an interval* $[a, b]$ *and be* _____ *on the interior* (a, b).

   Then there is _____ *such that* _____ $= f'(c)$.

2. A car is stopped at time $t = 0$. At time $t = 60$ minutes, the car has traveled 100 miles from its original position. This question will show you how to apply the MVT to show there must have been a point in time when the speed of the car was 100 mph.

   (a) What does the function f, to be used in the statement of the MVT, represent? Is it continuous?

   (b) What is the interval $[a, b]$ to be used for the MVT? What are the units of a, b?

   (c) Using the MVT, write in complete sentences why there is a point in time when the speed of the car was 100 mph.

3. Show that the point c guaranteed to exist by the MVT for $f(x) = x^2$ on $[a, b]$ is the arithmetic mean of a and b. (*Hint: the arithmetic mean of* $a_1, \ldots, a_n$ *is* $\frac{a_1+\cdots+a_n}{n}$)

4. Show that the point c guaranteed to exist by the MVT for $f(x) = \frac{1}{x}$ on $[a, b]$ is the geometric mean of a and b. (*Hint: the geometric mean of* $a_1, \ldots, a_n$ *is* $(a_1 \cdots a_n)^{1/n}$)

5. Let L_0 be the linear approximation of x^2 at 0, and L_k the linear approximation of x^2 at (k, k^2). Find the point (x, y) where L_0 intersects L_k.

6. Let $L_0(x)$ be the linear approximation of $2\arctan(x)$ at $x = 0$. Find $\lim_{x \to 0} \left[\dfrac{\sin(3x)}{L_0(x)} \right]$.

7. Suppose that f satisfies the hypotheses of the MVT on $[a, b]$.

   (a) Mathematically, what does the statement "f is increasing on (a, b)" mean?

   (b) Assume that f' is positive on (a, b). Using the MVT, show that f is increasing on (a, b).