

17 March 2016

1. **Warm up:** Answer the following questions with True / False.

- (a) If  $f(x)$  is a linear function, then the linear approximation of  $f$  at any point is  $f$  itself.
- (b) The linear approximation to  $x^2$  at  $x = 0$  is  $y = 0$ .
- (c) The linear approximation to  $f$  at a point  $a$  for which  $f'(a) = 0$  is  $y = 0$ .
- (d) For a positive function  $f$ , minimizing / maximizing  $f(x)$  is the same as minimizing / maximizing  $f^2(x)$ .

2. Consider the function  $f(x) = \cos(x)$ .

- (a) For any angle  $\theta$ , give the linearization of  $f$  at  $x = \theta$  and  $x = \theta + 2\pi$ .
- (b) Draw  $f$  and the two linearizations you found above for  $\theta = \pi/6$  on  $0 \leq x \leq 4\pi$ .



- (c) What is the difference in the  $y$ -intercepts of the two linearizations from part (b)?
- (d) What is the difference in the  $y$ -intercepts of the two linearizations from part (a)?
- (e) What does this say about the slope of  $f$  and how often it repeats?

3. Consider the function  $f(x) = \sqrt{x}$ .

(a) Find the point  $(x, y)$  on the graph of  $f$  nearest to  $(4, 0)$ . *Hint: Recall 1.(d) above.*

(b) Find the linear approximation to  $f$  at this point.

(c) Find the equation of the line through this point and  $(4, 0)$ .

(d) What is the relation between the slopes of the two lines you found in parts (b) and (c)?