

8 March 2016

1. **Warm up:** Consider the following problem:

Find the maximum and minimum values of the function $f(x) = 5x^5 + 4x^4 - 3x^3 + x - 10$.

Do not solve the problem! Instead list the steps you would take to solve the problem.

2. **Graphing functions:** Consider the function

$$f(x) = \frac{x^3 - 4x^2 + 3x}{x^2 + 3x - 10}.$$

- (a) What are the x - and y -intercepts of this function?
- (b) What are the asymptotes of the function?
- (c) What are the maxima and minima of the function?
- (d) Graph the function.
3. **Inverses of functions:** Knowing functions and their inverses, simplify completely the following expressions. Be careful with the first two!
- | | |
|--------------------------------------|---|
| (a) $\sqrt{x^2}$ | (d) $\ln(e^{x^2} e^{2x} e^4)$ |
| (b) $e^{\ln(x)}$ | (e) $\arccos(\operatorname{arccot}(x))$ |
| (c) $\tan(\operatorname{arcsec}(x))$ | (f) $\arcsin(\cos(\operatorname{arccot}(x)))$ |

4. **Related rates:** A rocket blasts off from ground level, and its height t seconds after liftoff is given by $p(t) = t^2$ feet. It lifts off from the center of a cylindrical tube measuring 40 feet in diameter and 50 feet in height. As the rocket rises, it glows brightly from the burning fuel, but there is an area around the tube outside that does not see this glow. At what rate is this area not reached by the glow decreasing at $t = 8$ seconds?

5. **Differentiation rules:** Differentiate the following functions, with respect to z .

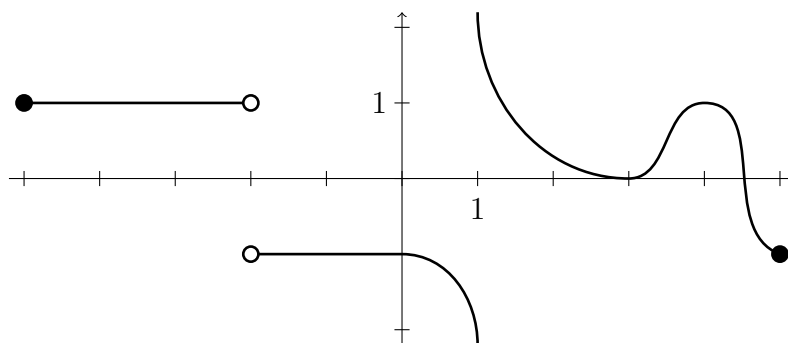
(a) $\sqrt{z+2} \ln(z^3+2z)$

(c) $2 \cos(z)^{z+1}$

(b) $\ln(\arctan(3^z)/\pi) + 22$

(d) $10z^2 + (e^{z+3}/e^3)^{z^2}$

6. **Concavity:** Below is the graph of the derivative f' of a continuous function f on $[-5, 5]$.



(a) On what intervals is f increasing? Decreasing?

(b) On what intervals is f concave down? Concave up?

(c) What are the x -values of the local minima and maxima of f ?

(d) What are the x -values of the points of inflection of f ?

(e) On top of the graph above, draw a possible continuous function f that could have the graph as derivative.

7. **Cost and revenue:** Let $C(p) = -p^2/10 + 100p + 800$ be the cost of producing p pillows.

(a) Find the average cost and marginal cost functions.

(b) Determine the average and marginal cost for producing 500 pillows.

(c) What is the difference in cost of producing 300 pillows and 400 pillows?