ESP Math 179 **Spring 2016**

Worksheet 14

25 February 2016

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1. **Warm up:** Consider the function $f(x, y, z) = z^2y^2 + x^2yz$, and assume that all variables x, y, z depend on time t. Take the derivative of f with respect to t and solve for:

   (a) $\dfrac{dx}{dt} =$

   (b) $\dfrac{dy}{dt} =$

   (c) $\dfrac{dz}{dt} =$

2. A 12 foot ladder is leaning against a wall when it starts to slip. The top of the ladder falls vertically toward the ground at a constant rate of 3 ft/sec.

   (a) Draw a diagram of this situation. Label and describe all variables you are using.

   (b) Give expressions (not numerical, just with symbols) for:

   i. the speed at which the top of the ladder is falling toward the ground

   ii. the speed at which the bottom of the ladder is moving away from the wall

   (c) Evaluate parts i. and ii. above numerically when the top of the ladder has fallen 5 feet.

   (d) Give an expression for the angle the ladder makes with the ground, in terms of previously defined variables.

   (e) How fast is the angle made by the ground and the ladder changing when the top of the ladder is 2 feet from the ground?

3. Ohm's law describes the relationship between the voltage V across a resistor, the electrical current I passing through the resistor, and a quantity R known as the resistance. The law is $V = IR$.

   (a) Suppose the current is increasing at a rate of 3 amps/sec while the resistance is holding steady at 4 ohms. How quickly is the voltage across the resistor increasing?

   Now suppose the voltage across the resistor is held constant at 20 volts, while the resistance is steadily increased at a rate of 4 ohms/sec.

   (b) What is the current through the resistor when the resistance reaches 10 ohms?

   (c) At what rate is the current changing at that time? Is it increasing or decreasing?

4. Suppose the Earth is being pulled into a black hole. The pull of gravity F from the black hole on earth is given by

$$F = \frac{GmM}{r^2},$$

   where m is the mass of the Earth, M the mass of the black hole, r the distance between the two, and G a constant that make the units work out. Suppose the mass of the black hole is increasing at a rate of one Earth mass per year, and the distance between the black hole and the earth decreases by one astronomical unit (distance between earth and sun) per year. How fast is the force of gravity changing when the Earth is 1/10 astronomical units away from the black hole?