Worksheet 13

23 February 2016

1. (a) Give an equation for which you need to use implicit differentiation to find the derivative.

   (b) How many first derivatives does a function have? How many functions have it as a first derivative?

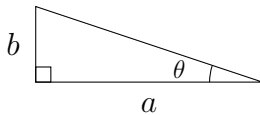   (c) Which of the following relations are functions of x?

   i. $y + 3 = x - 4 + 2$ ii. $e^y = e^e + e^y$ iii. $(y - 2)^2 + (x - 1)^2 = 5$

2. In this question you will find the derivatives of inverse trigonometric functions.

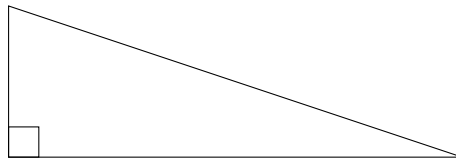   (a) Given the right triangle below, find the sine, cosine, and tangent of θ.

$\sin(\theta) =$

$\cos(\theta) =$

$\tan(\theta) =$

   (b) Given that $\arcsin(p) = \alpha$, construct a right triangle (give all the side lengths and angles) with one angle α and one side length p.

   (c) Using the triangle you constructed above, find the following values:

$\sin(\arcsin(p)) =$ $\csc(\arcsin(p)) =$

$\cos(\arcsin(p)) =$ $\sec(\arcsin(p)) =$

$\tan(\arcsin(p)) =$ $\cot(\arcsin(p)) =$

   (d) Using the identity $\sin(\arcsin(x)) = x$ and the chain rule, find the derivative of $\arcsin(x)$.
   Hint: Construct a right triangle with one side length x and hypotenuse length 1.

3. Using the technique from Question 2, find the derivatives of the following functions.

(a) $\arccos(x)$

(d) $\text{arcsec}(x)$

(b) $\arctan(x)$

(e) $\text{arccot}(x)$

(c) $\text{arccsc}(x)$

(f) $\arccos(\text{arccot}(x))$