Worksheet 12

18 February 2016

1. **Warm up:** Given the relation $4xy^2 + 2y - 10 = \cos(x^2 + y^2)$, find:

   (a) $\dfrac{dx}{dx} =$

   (b) $\dfrac{dx}{dy} =$

   (c) $\dfrac{dy}{dx} =$

   (d) $\dfrac{dy}{dy} =$

2. Find the derivatives of $f(x)$, $f(f(x))$, and $f(f(f(x)))$ for the following functions f.

   (a) $f(x) = x^2$

   (b) $f(x) = \sqrt{x}$

   (c) $f(x) = e^x$

3. Complete the following table, filling in yes/no in the boxes if the given function has/doesn't have the given property on the whole real line.

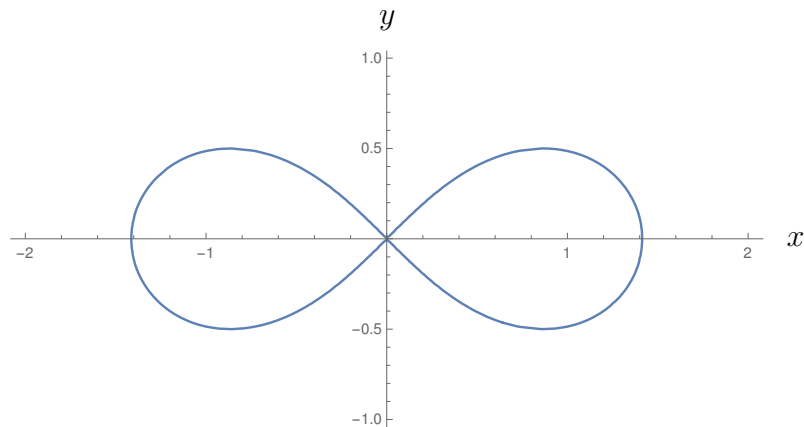|	e^x	$\ln(x)$	$1/x$	$\sin(x)$	$\tan(x)$	0	1	x^2	x^3	$\arctan(x)$	$|x|$
continuous											
differentiable											
constant											
odd											
even											
invertible											
increasing											
has VA's											
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periodic											
rational											

3. Below is the graph of $(x^2 + y^2)^2 = 2x^2 - 2y^2$.

Just by looking at the graph, answer the following questions.

(a) How many points on the graph are there for which $\dfrac{dy}{dx} = 0$?

(b) Choose any real number c.

   i. How many points on the graph are there for which $\dfrac{dy}{dx} = c$?

   ii. How many points on the graph are there for which $\dfrac{dx}{dy} = c$?

4. Consider the function $y(x)$ given by $y = x^2y^3 + x^3y^2$, with $y(1) = 1$.

   (a) What is the derivative of $y(x)$ at $x = 1$?

   (b) What is the derivative of $e^{y(x)}$ at $x = 1$?