ESP Math 179

Worksheet 11

Spring 2016

16 February 2016

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1. **Warm up:** Identify and correct the mistakes in the following solutions.

   (a) $\lim_{x\to 9} \dfrac{\sin(x-9) - 2^{x-7}}{x^2+1} = \lim_{x\to 9} \dfrac{\sin(9-9) - 2^{9-7}}{9^2+1} = \lim_{x\to 9} \dfrac{\sin(0) - 2^{-2}}{18+1}$, so $\lim_{x\to 9} = \dfrac{5}{19}$

   (b) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{(x-9)} = \dfrac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)} = \dfrac{-3}{-3(\sqrt{x}+3)} \quad\to\quad \sqrt{9}+9 \quad\to\quad 12$

2. Recall the rules for manipulating exponentials and logarithms and the chain rule.

   (a) Use the fact $e^{\ln(x)} = x$ to compute $\dfrac{d}{dx}\ln(x)$.

   (b) Let a be a positive number. Simplify $e^{x\ln(a)}$, that is, write an equivalent expression without e in it using laws of logarithms.

   (c) Evaluate $\dfrac{d}{dx}a^x$.

   (d) Evaluate $\dfrac{d}{dx}x^x$.

   (e) If we have a function f that has an inverse $f^{-1} = g$, we can write $f(g(x)) = x$. Use this and the chain rule to show

   $$g'(x) = \frac{1}{f'(g(x))}.$$

   Explain how part (a) is a special case of this equality.

3. Consider the function $f(x) = e^{-x^2}$.

   (a) Find the tangent line to $f(x)$ at $x = 0$.

   (b) Find all asymptotes of f.

   (c) Where are the slopes of the tangent lines to $f(x)$ positive, negative, and zero?

   (d) Sketch a graph of $f(x)$.

4. Assume $g(x) > 0$ for all x and let a be a positive number. Find $f'(x)$ for each of the following functions.

   (a) $f(x) = g(x)(x - a)$
   (b) $f(x) = g(a)(x - a)$
   (c) $f(x) = g(x + g(x))$
   (d) $f(x) = \dfrac{g(x)}{x - a}$
   (e) $f(x) = g(xg(a))$

5. Use the product and chain rule to discover the quotient rule. That is, using only the product and chain rule, compute

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right).$$