ESP Math 179

Worksheet 10

Spring 2016

11 February 2016

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1. (a) Consider a circle C of radius r.

    i. What is the circumfrence of C?

    ii. What is the area of C?

    iii. What is the derivative of the area of C, with respect to r?

   (b) Consider a sphere S of radius r.

    i. What is the surface area of S?

    ii. What is the volume of S?

    iii. What is the derivative of the volume of S, with respect to r?

   (c) Recall the derivative as a difference quotient is $\lim_{x \to a} \left[\frac{f(x)-f(a)}{x-a} \right]$. Interpret graphically the numerator of this quotient with the functions from (a)iii. and (b)iii.

   (d) What pattern do you see emerging? Explain what is happening.

2. Define two functions $\sinh(\theta) = \frac{e^{\theta}-e^{-\theta}}{2}$ and $\cosh(\theta) = \frac{e^{\theta}+e^{-\theta}}{2}$, called "hyperbolic sine" and "hyperbolic cosine."

   (a) Show that $\frac{d}{d\theta}\cosh(\theta) = \sinh(\theta)$ and $\frac{d}{d\theta}\sinh(\theta) = \cosh(\theta)$.

   Similar forms may be given for $\sin(\theta) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$ and $\cos(\theta) = \frac{e^{i\theta}+e^{-i\theta}}{2}$.

   (c) Using the above facts and the sum/difference formula for $\sin(a \pm b)$ and $\cos(a \pm b)$, express $\sin(x+iy)$ and $\cos(x+iy)$ using the functions given, without imaginary arguments.

3. Assume f is differentiable with the following value for f and f' as given below.

x	$f(x)$	$f'(x)$
0	3	-1
1	5	0
2	-2	3
3	6	1

   Let $g(x) = x^2 - 3x + 2$. For each function below, calculate the derivative at the given point.

   (a) $f(x) + g(x)$ at $x = 0$

   (b) $\dfrac{f(x)}{g(x)}$ at $x = 1$

   (c) $f(x)g(x)$ at $x = 2$

   (d) $\dfrac{f(x)g(x)}{f(x)+g(x)}$ at $x = 3$

   (e) $f(g(x))$ at $x = 0$

   (f) $f(g(x))$ at $x = 1$

   (g) $g(f(x))$ at $x = 2$

   (h) $g(f(x))$ at $x = 3$