ESP Math 179 | **Spring 2016**

Worksheet 9

9 February 2016

1. **Squeeze theorem:** For all $x \geqslant 4$, you are given that $x \leqslant x\ln(x) \leqslant e^x$. Use this identity and the squeeze theorem to find
$$\lim_{x\to\infty}\left[\frac{\ln(x)}{e^x}+1\right].$$

2. **Intermediate value theorem:**

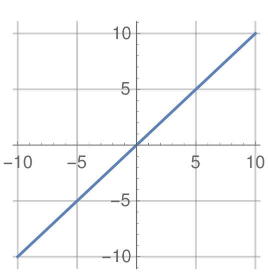
   (a) Use the IVT to show that the function $2\sin(2x+\pi)+10x-2$ has a root on the interval $[-\pi,\pi]$.

   (b) Consider the polynomial $f(x)=ax^3+bx^2+cx+d$ for positive real numbers a,b,c,d. Use the IVT to show that f has a root (that is, a point $x_0\in\mathbf{R}$ for which $f(x_0)=0$).

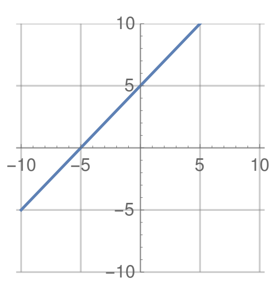
3. **Graphs of derivatives:** Given the graphs and their equations below, draw (without using a calculator, if possible) and give the equations of:

   (a) the derivative of each function,

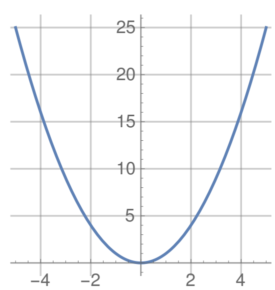
   (b) a function that could have the given function as derivative.



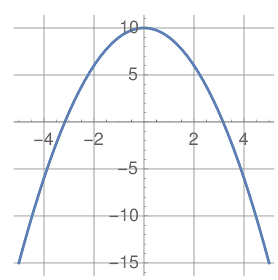
$y=x$



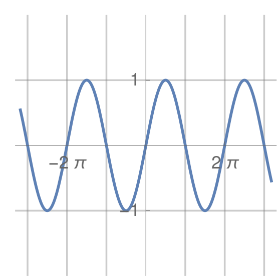
$y=x+5$



$y=x^2$



$y=-x^2+10$



$y=\sin(x)$

4. **Limits:** Evaluate the following limits.

(a) $\lim_{x\to 0}\left[\frac{\sin(4x)}{2x}\right]$

(c) $\lim_{x\to\infty}\left[\frac{\sin^2(x)-2x^3}{5x^3+2}\right]$

(b) $\lim_{x\to 3}\left[\sqrt[4]{\frac{7e^{3-x}+x^2}{\cos(\pi x)+2}}\right]$

(d) $\lim_{x\to 1}\left[\frac{1-x}{1-\sqrt{x}}\right]$

5. **Derivatives:** Take the derivative of the following functions with respect to x. Use the limit definition of the derivative for the first one.

(a) $4x^2 - 2x + 5/2$

(c) $\frac{\sin(2x)}{3x^2+\tan(x+1)}$

(b) $(e^{2x-5}-2)\left(\sqrt{6x+\sqrt{x}}-\frac{1}{x}\right)$

(d) $\frac{1+\frac{e^x}{\ln(x)}}{\frac{4x^2}{\cos(x)}-2x}$

6. **Tangent lines:**

(a) Find the equations of the tangent lines of $g(z) = \ln(x^2)/x$ at $x = -1, 2, e$.

(b) Find all the points (x, y) where the tangent lines of the function $f(x) = x^3 - x^2 - 4x + 4$ have slope 1.