ESP Math 179 | # Worksheet 7 | **Spring 2016**

2 February 2016

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1. **Warm up:** Consider the following question:

   "Use the IVT to show that if a man walks 4 miles in 1 hour, then at some time he must have walked exactly π miles."

   Which of the following answers are correct and which are not? Why?

   (a) Since $\pi \in (0, 4)$, and the man started walking at $t = 0$ hours and finished walking at $t = 1$ hour, there must be some point in time during this hour at which the man had walked exactly π miles.

   (b) If the distance the man has walked from the starting point is a continuous function of time on the interval $(0, 1)$, and $\pi \in (0, 4)$, then there is a $c \in (0, 1)$ such that at c hours after starting the man has walked π miles.

   (c) Note that the distance walked from time 0 is a continuous function of time (in hours) on the interval $(0, 1)$. Since $\pi \in (0, 4)$, by the IVT the must be some $c \in (0, 1)$ such that at c hours after starting the man had walked exactly π miles.

2. None of the limits below are correct for using as the derivative of $g(z)$. Indicate what is wrong with each one.

   (a) $\lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$ (b) $\lim_{z \to h} \left[\frac{g(z+h) - g(z)}{h} \right]$ (c) $\lim_{h \to 0} \left[\frac{g(z-h) + g(z)}{h - 0} \right]$

3. Identify for which x-values the following function are not continuous and for which x-values they are not differentiable. Draw the graph of each function (except the last one).

   (a) $y = |x|$

   (b) $y = |x|/x$

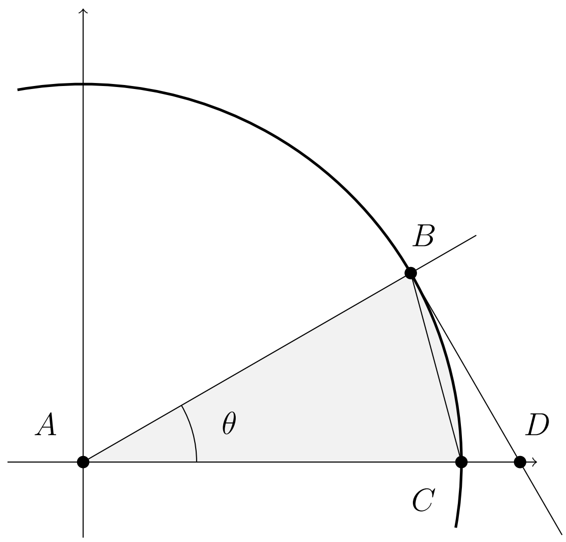
   (c) $y = |\sin(x)|$

   (d) $y = 1/x$

   (e) $x = 5$

   (f) $y = \sqrt[3]{x}/(x - 1)$

4. Consider the unit circle (circle of radius 1) in the first quadrant, as below.



In terms of $\sin(\theta)$ and $\cos(\theta)$:

(a) Express the area of the triangle ABC.

(b) Express the area of the triangle ABD.

(c) Given that the area of the sector (shaded area) ABC is $\frac{1}{2}\theta$, and the obvious inequality

$$(\text{area of triangle } ABC) \leqslant (\text{area of sector } ABC) \leqslant (\text{area of triangle } ABD),$$

prove the inequality

$$1 \leqslant \frac{\theta}{\sin(\theta)} \leqslant \frac{1}{\cos(\theta)}.$$

(d) Use part (c) and the squeeze theorem to evaluate $\lim_{\theta \to 0} \left[\frac{\sin(\theta)}{\theta} \right]$.

5. Using Question 4. above and the expression for $\sin(\alpha + \beta)$, find the derivative of $\sin(x)$. Use the limit definition!

6. **Bonus:** Using the definition of derivative you learned in class (limit definition), find the derivatives of the following functions.

(a) $f(x) = \frac{1}{x^2}$ (b) $g(y) = \sqrt{y}$ (c) $h(z) = \frac{2}{\sqrt{2z+1}}$