ESP Math 179 **Spring 2016**

Worksheet 5

26 January 2016

These are some definitions you should know:

polynomial function: A function $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, with $a_i \in \mathbf{R}$ for all i

rational function: A function $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomials, and $q(x) \neq 0$

exponential function: A function $f(x) = ab^x$, with $a, b \in \mathbf{R}$

1. **Warm up:** Identify each function below as polynomial, rational, exponential, or neither.

   (a) $f(x) = 2x + 2$

   (b) $g(x) = 4x^{-2}$

   (c) $h(x) = 0$

   (d) $k(z) = z + z^2 + z^3 + z^4 + \cdots$

   (e) $p(y) = y^y$

   (f) $q(w) = 2^w 4^w$

   (g) $r(t) = \dfrac{\frac{9^t(3t^2+2t-10)}{4t^2-1}}{3^{2t}}$

   (h) $s(t) = e^{t+5}$

2. Find the horizontal asymptotes of the following functions.

   (a) $f(x) = \dfrac{x}{2x - \sqrt{x^2-1}}$ (b) $g(x) = \dfrac{|2x-1|}{x+1}$ (c) $h(y) = \dfrac{\arctan(y)}{\pi} - 1$

3. Create functions with the asymptotes at the given lines.

   (a) $y = 0$

   (b) $y = 1$ and $x = 0$

   (c) $y = -5$, $x = -3$ and $x = 6$

   (d) $y = -1$ and $y = 1$

   (e) $x = 2k$ for every integer k (that is, for all $k = \ldots, -2, -1, 0, 1, 2, \ldots$)

4. Find the following limits, if they exist.

(a) $\lim_{x\to\infty}\left[\dfrac{3x^5-2x}{x^5+3x^4}\right]$

(b) $\lim_{x\to-\infty}\left[\dfrac{x^5+7x^3-10}{x^7+14x^6}\right]$

(c) $\lim_{x\to\infty}\left[\dfrac{x^2+3x-2}{\sqrt{3x^4+5x^3-2x+1}}\right]$

(d) $\lim_{x\to-\infty}\left[\sqrt{\dfrac{2x^2+2}{5x^5-3x^3+2x^2+2}}\right]$

(e) $\lim_{x\to-\infty}\left[x+\sqrt{x^2+3x}\right]$

(f) $\lim_{x\to\infty}\left[\dfrac{\cos(x)}{\sqrt{x}}+2\right]$

5. **Bonus:** Imagine a rope around the Earth at the equator. Add a 20 meter segment to the rope. The new rope is held in a circular shape centered at the Earth. How far off the ground is the new rope? Assume the Earth is a perfect sphere.