ESP Math 179 # Worksheet 2 **Spring 2016**

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1. Answer the following questions with either "True" or "False." For those you answer "False," try to find a counterexample.

   (a) If $\lim_{x\to 0} [f(x)] = 1$, then $\lim_{x\to 0} [f(x) - 1] = 0$.

   (b) As long as $\lim_{x\to 0} [f(x)]$ exists, $\lim_{x\to 0} [2f(x)] \geqslant \lim_{x\to 0} [f(x)]$.

   (c) For any two functions f and g, $\lim_{x\to 5} [f(x)] + \lim_{x\to 5} [g(x)] = \lim_{x\to 5} [f(x) + g(x)]$.

   (d) If $\lim_{x\to -2^+} [f(x)] \neq \lim_{x\to -2^-} [f(x)]$, then $f(-2)$ is not defined.

   (e) If $\lim_{x\to 3} [h(x)]$ does not exist, then h does not have a tangent line at $x = 3$.

   (f) If h does not have a tangent line at $x = 3$, then $\lim_{x\to 3} [h(x)]$ does not exist.

2. Draw examples of functions with the given properties below.

   (a) $k(x)$ such that $\lim_{x\to 0^-} [k(x)] \neq \lim_{x\to 0^+} [k(x)] \neq k(0)$.

   (b) $m(x)$ such that $\lim_{x\to -2^-} [m(x)] = m\left(\lim_{x\to -2^+} [m(x)]\right)$.

   (c) $h(x)$ such that $\lim_{x\to -1} [5h(x)] = \lim_{x\to 1} [h(x) + 2]$, $\lim_{x\to 0} [h(h(x))]$ does not exist, and $h(0) = 0$.

3. The floor function $f(x) = \lfloor x \rfloor$ gives the largest integer less than or equal to x.

   (a) Where is f defined?

   (b) Where does $\lim_{x\to a} [f(x)]$ exist?

   (c) Where does $\lim_{x\to a^+} [f(x)]$ exist?

   (d) Where does $\lim_{x\to a^-} [f(x)]$ exist?

4. (a) Find two different functions that have a point where the limit does not exist (the point may be different for each function).

   (b) Multiply the two functions you found in part (a) together.

      i. Does this function have points where the limit does not exist? If yes, where?

      ii. Can you come up with two functions in part (a) such that their product does not have any points where the limit does not exist?

   (c) Repeat part (b) with addition instead of multiplication.