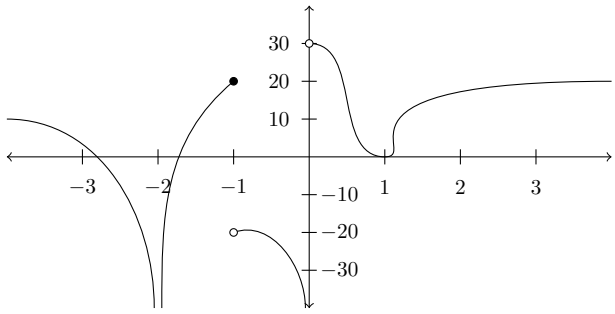
SURPRISE FINAL EXAM

Dec. 1, 2015

Complete the exam below on your own. You have two hours to finish it.

1. A balloon rises at a rate of 1 ft/sec from a point on the ground 60 ft away from an observer. Find the rate of change of the angle of elevation to the balloon from the observer when the balloon is 80 ft above the ground.

2. Use the graph shown here to answer the questions below.



   (a) $\lim_{x\to\infty} f(x)$

   (b) $\lim_{x\to 1^-} f(x)$

   (c) $\lim_{x\to -1} f(x)$

   (d) $\lim_{x\to -2^-} f(x)$

   (e) $\lim_{x\to 0^+} f(x)$

   (f) $f(-1)$

   (g) $f(0)$

3. Evaluate the following limits.

   (a) $\lim_{x\to 0^+} \dfrac{5-\sqrt{25+x}}{x\sqrt{25+x}}$

   (b) $\lim_{x\to 3} \dfrac{x^3-9x}{3-x}$

   (c) $\lim_{x\to 1} \dfrac{1+\cos(\pi x)}{2-2x+2\ln(x)}$

   (d) $\lim_{x\to 3} \dfrac{-3}{3+x}$

4. Find $\frac{dy}{dx}$ for the following functions. Do **NOT** simplify your answer.

   (a) $y = \dfrac{7^{7x}+\ln(3x^2+2)}{3-\csc(5x)+\frac{1}{\sqrt{x}}}$

   (b) $2x^3y^2+4x^2 = 3y^3-5x\ln(y)$

5. Using the limit definition of the derivative, find the derivative of $f(x) = 3x^2+4x-5$.

6. Answer the following questions using the function $f(x) = \dfrac{1}{4}x^4+4x^3+50$ on the interval $[-15,15]$.

   (a) On what intervals is the function **increasing**?

   (b) On what intervals is the function **concave down**?

   (c) Give the exact values of the **local extrema** and specify where they occur. Be sure to indicate whether the extrema are minima or maxima or neither.

   (d) Give the exact values of the **inflection points** and specify where they occur.

   (e) Give the exact values of the **absolute extrema** and specify where they occur. Be sure to indicate whether the extrema are minima or maxima or neither.

7. Compute the following definite/indefinite integrals.

   (a) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}} + 6x^2 + 5 \, dx$

   (b) $\displaystyle\int \frac{x}{\sqrt{x-10}} \, dx$

   (c) $\displaystyle\int \frac{1}{x^2+2x+10} \, dx$ (complete the square)

   (d) $\displaystyle\int_0^1 e^{x+e^x} \, dx$

8. (a) Find the equation of the tangent line to $f(x) = x^3 \ln(x)$ at $x = 1$.

   (b) Find the linear approximation to $g(x) = -x^3 \ln(x)$ at $x = 1$.

9. Give the expression for the right Riemann sum of n rectangles for the function $f(x) = x^3$ on the interval $[1, 4]$. Draw a picture of this when $n = 3$, is this an under or over estimate?