ESP Math 179 **Fall 2015**

Worksheet 26

24 November 2015

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1. **Warm up:** Answer the following questions with True / False.

   (a) The antiderivative and indefinite integral of a function are the same thing.

   (b) An even function cannot be an antiderivative of an odd function.

   (c) The definite integral of an odd function must be 0.

2. Take the derivative, and indefinite integral, with respect to x, of the following functions.

   (a) x

   (b) e

   (c) x^e

   (d) e^x

   (e) e^e

   (f) ee^{ex}

3. Given that $\int_0^{\pi/2} \cos(\theta) - 2\sin(\theta)d\theta = -1$, compute $\int_{\pi/2}^{0} 4\cos(\theta) - 8\sin(\theta)d\theta$.

4. Let

$$f(x) = \begin{cases} -2 & \text{if } x < 1, \\ 5 - 2x, & \text{if } x \geqslant 1. \end{cases}$$

   Graph $f(x)$ and compute $\int_{-2}^{4} f(x)dx$ and $\int_{-2}^{4} |f(x)|dx$.

5. Consider the function $g(t) = \int_0^t f(x)dx$. In this question you will show g is continuous.

   (a) What is the definition of continuity? That is, describe what it means for a function to be continuous at a point t_0

   (b) Give an expression for $|g(t) - g(t_0)|$.

   (c) Suppose that f is bounded by a value M. Use the fact that $\left|\int_a^b h(x)dx\right| \leqslant \int_a^b |h(x)|dx$ to show that your answer above is bounded by a finite number.

   (d) Take the limit as $t \to t_0$ to show that $g(t) \to g(t_0)$.

6. (a) Why is the integral of $\sin(x)$ on $[-2\pi, 2\pi]$ equal to 0?

   (b) Why is the integral of $\sin(x + \pi/2)$ on $[-5\pi/2, 3\pi/2]$ equal to 0?

   (c) Why is the integral of $\sin(x) + 1$ on $[-5, 5]$ not equal to 0?