

19 November 2015

1. **Warm up 1:** Fill in the blanks with  $\leq$ ,  $\geq$ , or  $=$ .

$$\int_a^b f(x) - g(x) dx \quad \_\_\_ \quad \left| \int_a^b f(x) - g(x) dx \right| \quad \_\_\_ \quad \int_a^b |f(x) - g(x)| dx \quad \_\_\_ \quad \int_a^b |f(x)| + |g(x)| dx$$

**Warm up 2:** Find functions  $f(x)$ ,  $g(x)$ , so that all of the integrals above are different.

2. Which of the following expressions do not make mathematical sense, and why?

(a)  $\int_{-2}^{-4} 4y^2 + 5y$

(b)  $\iiint abc da db dc$

(c) Evaluate  $\int g(z) \setminus dz$ .

(d)  $\lim_{n \rightarrow \infty} \left[ \int_{-n}^n \frac{1}{x^2} dx \right] = \left( \left( \frac{-1}{n} + C \right) - \left( \frac{-1}{-n} + C \right) \right) = \frac{-2}{n} = \frac{-2}{\infty} = 0$

3. Evaluate the following integrals using the fundamental theorem of calculus.

(a)  $\int_0^1 x + 1 dx$

(c)  $\int_0^1 (x + 1)^3 dx$

(b)  $\int_0^1 (x + 1)^2 dx$

(d)  $\int_0^1 (x + 1)^4 dx$

(e) Let  $f(n) = \int_0^1 (x + 1)^n dx$ . In terms of  $f(n)$ , can you guess what  $f(n + 1)$  will be?

4. Compute the derivative, with respect to  $t$ , of the following functions.

(a)  $g(t) = \int_0^{t^2} e^{-x} dx$

(c)  $\phi(t) = \int_a^{t^4} f(x) dx$

(b)  $h(t) = \int_7^{t^3} \sec^2(x) dx$

(d)  $\psi(t) = \int_a^{b(t)} f(x) dx$

5. So far you have seen finite bounds for definite integrals. This question will ask you consider infinite bounds.

(a) Explain what you think  $\int_0^{\infty} f(x) dx$  means. Use symbols and expressions you know.

(b) Apply part (a) to guess what an answer for  $\int_0^{\infty} e^{-x} dx$  would be.