

17 November 2015

1. **Warm up:** Recall the general forms of the n th Riemann sum of f over $[a, b]$. Identify each of the following expressions as either the left, right, or midpoint Riemann sums.

$$(a) \sum_{i=1}^n \frac{b-a}{n} f\left(a + (i-1) \cdot \frac{b-a}{n}\right)$$

$$(c) \sum_{i=1}^n \frac{b-a}{n} f\left(a + (i-1) \cdot \frac{b-a}{n} + \frac{b-a}{2n}\right)$$

$$(b) \sum_{i=1}^n \frac{b-a}{n} f\left(a + i \cdot \frac{b-a}{n}\right)$$

$$(d) \sum_{i=0}^{n-1} \frac{b-a}{n} f\left(a + i \cdot \frac{b-a}{n}\right)$$

2. (a) Give the n th midpoint Riemann sum of $f(x) = x^2 + x$ over $[0, 10]$. Do not simplify.

- (b) Simplify your previous answer, using the following identities:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

- (c) Take the limit of your previous answer, as $n \rightarrow \infty$.

- (d) Compute the definite integral $\int_0^{10} x^2 + x \, dx$ and compare it to your previous answer.

3. Graph $f(x) = 2 - |x|$ and compute $\int_0^2 f(x)dx$.

4. The area of a rectangle with height h and base b is a two variable function $A(h, b) = hb$.

(a) Explain the meaning of $A(2h, b) = 2A(h, b)$.

(b) How is this related to $\int_a^b 2f(x)dx = 2 \int_a^b f(x)dx$?