ESP Math 179 **Fall 2015**

Worksheet 22

10 November 2015

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1. **Warm up:** Let f be a function with an antiderivative F, and g a function with an antiderivative G. That is, $F'(x) = f(x)$ and $G'(x) = g(x)$. Answer the following questions with True / False.

   (a) If $f = g$, then $F = G$.

   (b) If F and G differ by a constant, then $f = g$.

   (c) If f and g differ by a constant, then $F = G$.

2. Find antiderivatives of the following functions.

   (a) $\dfrac{4x^{13} - 3x^{-4}}{x^2}$ (b) $\dfrac{\sin(\theta) - 1}{\cos^2(\theta)}$

3. (a) Let $k = n + 5$. Write $\displaystyle\sum_{n=0}^{50} n^2$ as a sum indexed over k

   (b) Combine $\displaystyle\sum_{j=0}^{n}(j^2 + 2j) - \sum_{j=1}^{n+1}(j - 1)$ into one sum.

   (c) Simplify the double sum $\displaystyle\sum_{a=0}^{n}\sum_{b=0}^{m}(a + b)^2$ given that $a = b - 3$.

4. Evaluate the following expressions.

   (a) $\int x^2 \, dx$

   (b) $\int \left(\sum_{n=0}^{20} nx^n \right) dx$

   (c) $\frac{d}{dx} \left(\sum_{n=0}^{20} nx^n \right)$

   (d) $\frac{d}{dx} \left(\sin(\theta) \int \cos(\theta) \, d\theta - \cos(\theta) \int \sin(\theta) \, d\theta \right)$

5. A base jumper dives off a cliff that is 200 meters high. Assume that acceleration due to gravity is 9.8 meters per second squared.

   (a) What is the velocity of the jumper as a function of time?

   (b) What is his position as a function of time?

   (c) He pulls his parachute open when he is 50 meters away from the ground. What is his speed at this instant?