ESP Math 179 | **Fall 2015**

Worksheet 21

5 November 2015

---

1. **Warm up:** Evaluate the following limits using l'Hôpital's rule.

   (a) $\lim_{x \to 1} \left[\frac{x^n - 1}{x - 1} \right]$ (b) $\lim_{z \to 0} \left[\frac{\tan(4z)}{\tan(7z)} \right]$ (c) $\lim_{x \to 0^+} [x^x]$

2. Suppose that $\lim_{x\to\infty} \left[\frac{f(x)}{g(x)}\right]$ is indeterminate of the form $\frac{0}{0}$. Rearrange the functions f and g to show that this limit may be viewed as:

   (a) indeterminate of the form $\frac{\infty}{\infty}$,

   (b) indeterminate of the form $0 \cdot \infty$.

3. The mean value theorem (MVT) says if f is continuous on $[a, b]$ and differentiable on (a, b), then there is at least one c between a and b such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.

   (a) Find a function f and an interval $[a, b]$ where there is only one such c.

   (b) Find a function f and an interval $[a, b]$ where there are exactly two such c's.

   (c) Find a function f and an interval $[a, b]$ where there are infinitely many such c's.

4. Evaluate the following limits using l'Hôpital's rule (and any other tools you know).

(a) $\lim_{x \to 2^+} \left[\frac{1}{x-2} - \frac{1}{\ln(x-1)} \right]$

(d) $\lim_{x \to 0^+} \left[(\cos(x) - 1)^x \right]$

(b) $\lim_{x \to 0^+} \left[(e^x + x)^{\frac{1}{4x}} \right]$

(e) $\lim_{x \to \pi/2^+} \left[\frac{\sec(x)}{1 + \tan(x)} \right]$

(c) $\lim_{x \to 0^+} \left[\frac{\sin(x) - x}{x^3} \right]$

(f) $\lim_{x \to 0^+} \left[\frac{2\ln(e^x - 1)}{\ln(3x)} \right]$