ESP Math 179 **Fall 2015**

Worksheet 20

3 November 2015

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1. **Warm up:** Fill in the blanks for the statement of the mean value theorem (MVT):

   Let f *be* _____ *on an interval* $[a, b]$ *and be* _____ *on the interior* (a, b).

   Then there is _____ *such that* _____ $= f'(c)$.

2. A car is stopped at time $t = 0$. At time $t = 60$ minutes, the car has traveled 100 miles from its original position. Apply the MVT to show there must have been a point in time when the speed of the car was 100 mph.

   (a) What does the function f, to be used in the statement of the MVT, represent?

   (b) What is the interval $[a, b]$ to be used in the MVT? What are the units of a, b?

   (c) Write the complete statement of the MVT in the context of this problem.

3. Show that the point c guaranteed to exist by the MVT for $f(x) = x^2$ on $[a, b]$ is the arithmetic mean of a and b. (*Hint: the arithmetic mean of* $a_1, \ldots, a_n$ *is* $\frac{a_1+\cdots+a_n}{n}$)

4. Show that the point c guaranteed to exist by the MVT for $f(x) = \frac{1}{x}$ on $[a, b]$ is the geometric mean of a and b. (*Hint: the geometric mean of* $a_1, \ldots, a_n$ *is* $(a_1 + \cdots + a_n)^{1/n}$)

5. Let L_0 be the linear approximation of x^2 at 0, and L_k the linear approximation of x^2 at (k, k^2). Find the point (x, y) where L_0 intersects L_k.

6. Suppose that f satisfies the hypotheses of the MVT on $[a, b]$.

   (a) Mathematically, what does the statement "f is increasing on (a, b)" mean?

   (b) Assume that f' is positive on (a, b). Using the MVT, show that f is increasing on (a, b).

7. Let $L_0(x)$ be the linear approximation of $2\arctan(x)$ at $x = 0$. Find $\lim_{x \to 0} \left[\dfrac{\sin^2(x)}{L_0(x)} \right]$.