

27 October 2015

1. **Warm up:** How many maxima and minima do each of the functions have on the given interval? You do not need to find the extrema, you just need to say how many there are.

(a)  $y = x^2$  on  $(-\infty, \infty)$

(d)  $y = \sin(x)$  on  $[0, 4\pi]$

(b)  $y = x(x - 5)(x + 5)$  on  $[-6, 6]$

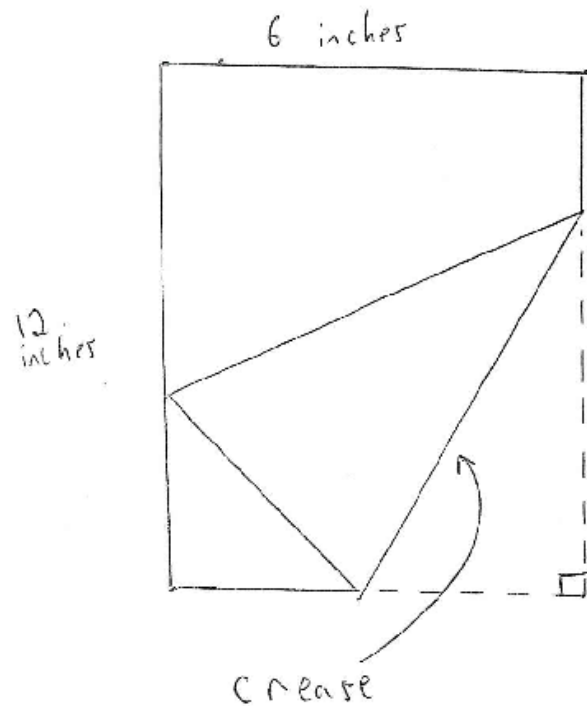
(e)  $y = e^x$  on  $[-100, 100]$

(c)  $y = \tan(x)$  on  $[-\pi/2, \pi/2]$

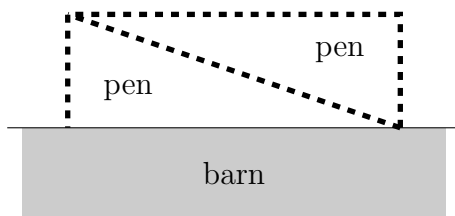
(f)  $y = \arctan(x)$  on  $(0, \infty)$

2. A rectangular piece of paper is 12 inches high and 6 inches wide. The lower right-hand corner is folded over to reach the left edge of the paper, as in the picture to the right.

What is the smallest possible length for the crease?

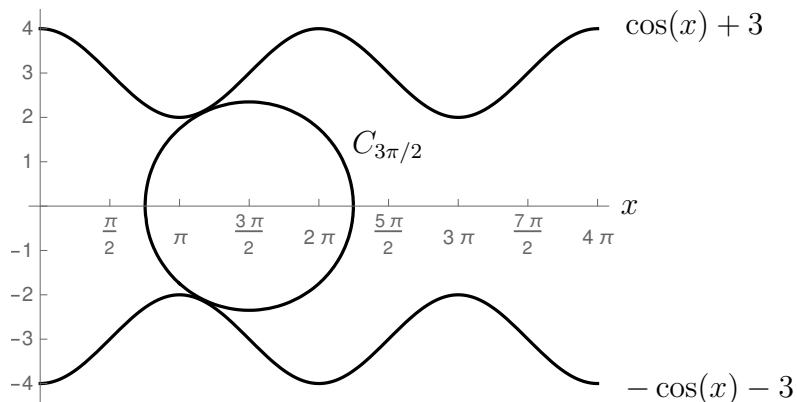


3. Two triangular pens are built against a barn. Two hundred meters of fencing are to be used for the three sides and the diagonal dividing fence as in the picture.



What dimensions of the pens maximize the area of the pens?

4. Consider a circle  $C_x$  centered on the  $x$ -axis and bounded by the functions  $\cos(x) + 3$  and  $-\cos(x) - 3$ . Given an  $x$ -value, the circle  $C_x$  is as large as possible, while still being bounded by the two functions, as in the picture below.



- (a) At what  $x$ -value(s) do you think the circle reaches its maximum size?
- (b) Find the equation of the circle  $C_\pi$ .
- (c) Given an  $x$ -value  $x_0$ , create an objective function  $f$  such that when optimized,  $f$  gives the radius of  $C_{x_0}$ .