Worksheet 17

22 October 2015

1. The volume of a telephone ringing, in decibels, t seconds after it starts ringing, is given by the piecewise function

$$v(t) = \begin{cases} 50 & \text{if } t \in [2k\pi, (2k+1)\pi)), \\ 50 \sin^2(t + \pi/2) & \text{if } t \in [(2k-1)\pi, 2k\pi), \end{cases}$$

for integers k.

   (a) Sketch the graph of the function $v(t)$ for $0 \leqslant t \leqslant 4\pi$.

   (b) Find the derivative of the function $v(t)$. Is the derivative continuous?

   (c) A man is 20 meters away from the telephone when it starts to ring. He is walking further away from the telephone at a rate of 5 meters/second. Assuming the speed of sound is 100 meters/second, how long after his phone starts ringing does the man hear that hotline bling?

2. Let $f(x)$ be an odd degree polynomial. Prove that f has at least one real root.

   Hint: What theorem have you been using to show functions have a zero? Use limits to show odd degree polynomials satisfy the hypothesis of this theorem.

3. Suppose we have a function $f(x)$ such that $f''(x) - 5f'(x) + 6f(x) = 0$. This is called a *differential equation* and is the main application of calculus to the real world.

   (a) You are given that $f(x) = ce^{rx}$ for some constants $c \neq 0, r$. Rewrite the equation above with this information.

   (b) Can you find what r is?

   (c) Can you find what c is?