Worksheet 15

15 October 2015

1. **Warm up:** Pretend you are a pirate on a ship. A cannonball is fired from a cannon on your ship toward another ship. You are tasked with determining if the cannonball will hit the other ship. Knowing that projectiles follow a parabolic path, write down three questions you would ask to solve this problem.

2. Two atoms are moving in a plane. The position of the first atom is given by $(x_1(t), y_1(t))$ and the position of the second atom is given by $(x_2(t), y_2(t))$.

   (a) What is the distance between the two atoms at time t?

   (b) How fast is the distance between the two atoms changing at time t?

   Suppose that $x_1(t) = x_2(t) = t$ and $y_1(t) = -y_2(t) = \cos(t)$.

   (c) Draw a diagram of the position of both atoms from $t = 0$ to $t = 4\pi$.

   (d) How fast is the distance between the two atoms changing at time t?

3. (a) Find the value of x that minimizes $f(x) = (x-1)^2 + (x-5)^2$.

   (b) Find the value of x that minimizes $f(x) = (x-a)^2 + (x-b)^2$.

   (c) Find the value of x that minimizes $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$.

   (d) **Bonus:** What do you think is the x-value that minimizes $f(x) = \sum_{i=1}^{n} (x - a_i)^2$?

4. A rectangle has its lower left corner at $(0,0)$ and its upper right corner on the circle of radius 2 centered at $(0,0)$, as in the picture below.
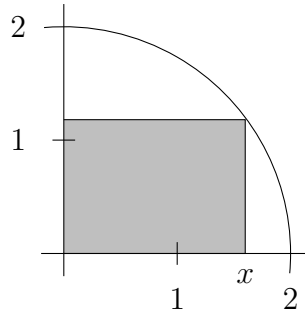
Find x between 0 and 2 that maximizes the area of the rectangle.

5. Consider the function $f(x) = \frac{6}{x^2+3}$.

   (a) Find where f reaches its largest and smallest values.

   (b) Find where the slopes of tangent lines of f are steepest and flattest.