ESP Math 179

Worksheet 14

Fall 2015

13 October 2015

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1. **Warm up:** Identify which of the functions below, given the domain, have any of the following: local min, local max, absolute min, absolute max.

   (a) $f(x) = x^2$ for $x \in [-3, 5]$

   (b) $g(y) = x^3$ for $y \in [-1, 10)$

   (c) $h(z) = \sin(z)$ for $z \in (-\pi/3, \pi/4]$

   (d) $k(w) = \arctan(w)$ for all $w \in \mathbf{R}$

   You should know exactly what these functions look like! If not, find out and make sure you know what they look like. They are very important.

2. Let $f(x)$ be a differentiable function. Explain how to find all the maxima and minima of f on a closed interval $[a, b]$.

3. Give an explanation, in your own words, of the following sentences.

   (a) "4 is a local maximum for $f(x)$ on $[-3, 2]$, which is reached at $x = 1$"

   (b) "π is an absolute minimum on $\mathbf{R}$ for the function $g(y)$ at all the even integers"

   (c) "the function $h(z)$ has no absolute maximum or minimum on $(10, 12)$"

4. A quadratic polynomial is a function of the form $f(x) = ax^2 + bx + c$ for some constants a, b, c. If f is a quadratic polynomial with two real roots r, s, then there always exists some constant k such that $f(x) = k(x - s)(x - r)$.

   (a) Find an example of a quadratic polynomial with two real roots and factor it.

   (b) For quadratic polynomial with two real roots r, s as above, show that $f'(r) = -f'(s)$.

   (c) Show that a critical point of a quadratic polynomial with two real roots occurs halfway between the roots.

5. Consider the function $f(x) = |x - 1| + |x + 1| + 1$.

   (a) Graph $f(x)$.

   (b) Find all maxima and minima of f, and label them as local and/or global.

   (c) **Bonus:** Repeat the same process for

$$g(x) = |x - 3| - |x - 2| + |x - 1| + |x + 1| - |x + 2| + |x + 3| + 1.$$