

24 September 2015

1. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Is f differentiable at $x = 0$?

2. (a) Consider a circle C of radius r .

i. What is the circumference of C ?

ii. What is the area of C ?

iii. What is the derivative of the area of C , with respect to r ?

- (b) Consider a sphere S of radius r .

i. What is the surface area of S ?

ii. What is the volume of S ?

iii. What is the derivative of the volume of S , with respect to r ?

- (c) What pattern do you see emerging? Try drawing the (graphical interpretation of the) numerator of the difference quotient from (a)iii. and (b)iii. to help you out.

3. Recall the product rule $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$. Do there exist functions f, g such that $(f(x)g(x))' = f'(x)g'(x)$?
4. The mayor of Chicago pays a monthly “fee” of 5000\$ to a local union boss to keep workers from causing problems. In October the mayor starts preparing for the next election, which is at the end of next May. Since the workers are becoming more restless the closer it comes to election time, the union boss asks 500\$ more from the mayor than each previous month, starting with October.
- (a) Write a function $f(t)$ for the amount of money in the union boss’s “secret special” bank account, the one in which the mayor deposits the money. The units t are months from this January, and assume he has k dollars in January (assume money is never spent from this account).
- (b) How fast is the amount of dollars in the union boss’s bank account changing per month?
- (c) How fast is the rate of deposited dollars per month in the union boss’s bank account changing, per month?
- (d) The state comptroller finds out about the mayor’s misdeeds in December. If the mayor must pay her as much as the union boss every month to keep her quiet, starting with December, how much money has the mayor wasted from this January to the election next May?