ESP Math 179

Worksheet 9

Fall 2015

22 September 2015

---

1. **Squeeze theorem:** For all $x \geqslant 4$, you are given that $x \leqslant x\ln(x) \leqslant e^x$. Use this identity and the squeeze theorem to find
$$\lim_{x\to\infty}\left[\frac{\ln(x)}{e^x}+1\right].$$

2. **Intermediate value theorem:** A wildfire in the prairies starts at 5AM on Tuesday, and is spreading at a constant rate of 10 square miles per hour. Firefighting crews begin extinguishing the fire at 11AM at a constant rate of 16 square miles an hour.

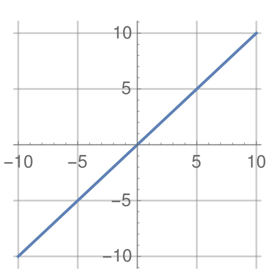
   (a) Do the firefighters ever extinguish all the flames? If so, when? If no, why not?

   (b) Using the intermediate value theorem, show that there is a time when the area burning is equal to the area already extinguished by the firefighters.

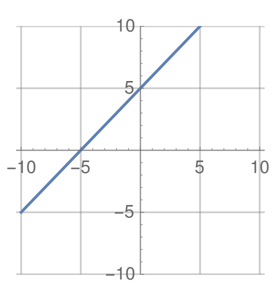
3. **Graphs of derivatives:** Given the graphs and their equations below, draw (without using a calculator, if possible) and give the equations of:

   (a) the derivative of each function,

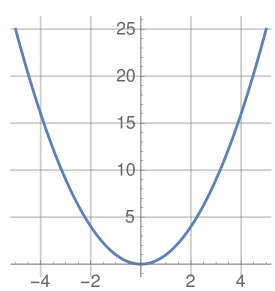
   (b) a function that could have the given function as derivative.



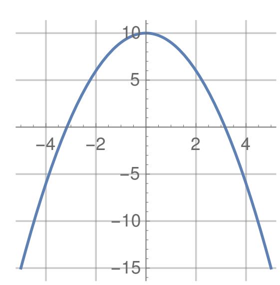
$y = x$



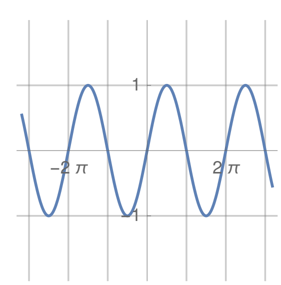
$y = x + 5$



$y = x^2$



$y = -x^2 + 10$



$y = \sin(x)$

4. **Calculating limits:** Evaluate the following limits.

(a) $\lim_{x \to 0} \left[\frac{\sin(4x)}{2x} \right]$

(b) $\lim_{x \to 3} \left[\sqrt[4]{\frac{7e^{3-x} + x^2}{\cos(\pi x) + 2}} \right]$

(c) $\lim_{x \to \infty} \left[\frac{\sin^2(x) - 2x^3}{5x^3 + 2} \right]$

(d) $\lim_{x \to 1} \left[\frac{1 - x}{1 - \sqrt{x}} \right]$

5. **Calculating derivatives:** Take the derivative of the following functions with respect to x.

(a) $(e^x - 2)(\sqrt{x} - \frac{1}{x})$

(b) $\frac{\sin(x)}{3x^2 + \tan(x)}$

(c) $\frac{1 + \frac{e^x}{\ln(x)}}{\frac{4x^2}{\cos(x)} - 2x}$