ESP Math 179

Worksheet 7

Fall 2015

15 September 2015

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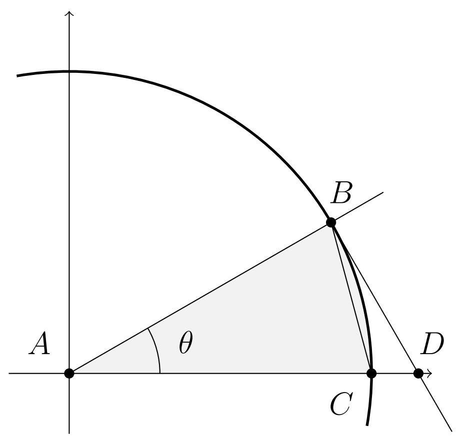
1. **Warm up:** Recall the definition of a 'differentiable function'. Use it to give examples of functions of the type described below.

   (a) not continuous and not differentiable at a point

   (b) continuous and not differentiable at a point

   (c) continuous and differentiable at a point

2. Consider the unit circle (circle of radius 1) in the first quadrant, as below.



   In terms of $\sin(\theta)$ and $\cos(\theta)$:

   (a) Express the area of the triangle ABC.

   (b) Express the area of the triangle ABD.

   (c) Given that the area of the sector (shaded area) ABC is $\frac{1}{2}\theta$, and the obvious inequality

   $$(\text{area of triangle } ABC) \leqslant (\text{area of sector } ABC) \leqslant (\text{area of triangle } ABD),$$

   prove the inequality

   $$1 \leqslant \frac{\theta}{\sin(\theta)} \leqslant \frac{1}{\cos(\theta)}.$$

   (d) Use the squeeze theorem to evaluate $\lim_{\theta \to 0} \left[\frac{\sin(\theta)}{\theta}\right]$.

3. (a) Graph $f(x) = \sin(x)/x$ on the interval $[-4\pi, 4\pi]$, with the help of a graphing calculator, if necessary.

   (b) Does the limit of f as x goes to 0 exist? Why or why not?

   (c) Is the function f continuous at $x = 0$? Why or why not?

4. (a) Write the definition of the derivative of a function f.

   (b) Using part (2) above and the expression for $\sin(\alpha + \beta)$, find the derivative of $\sin(x)$.

5. **Bonus:** Using the definiton of derivative you learned in class, find the derivatives of the following functions.

   (a) $f(x) = \dfrac{1}{x^2}$

   (b) $g(y) = \sqrt{y}$

   (c) $h(z) = \dfrac{2}{\sqrt{2z+1}}$