ESP Math 179

Worksheet 6

Fall 2015

10 September 2015

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1. **Warm up:** Identify where the following functions are continuous.

   (a) x

   (b) $|x|$

   (c) $\frac{1}{x}$

   (d) $\frac{1}{x-1}$

   (e) $\frac{x+1}{x-1}$

   (f) $|\sin(x)|$

   (g) $\tan(x)$

   (h) $\tan^{-1}(x)$

   (i) e^x

   (j) $\ln(x)$

2. Use the intermediate value theorem to show that the function $f(x) = xe^{-x} + \frac{1}{2}$ has a zero on the interval $[-5, 5]$. Write down a complete sentence for your answer.

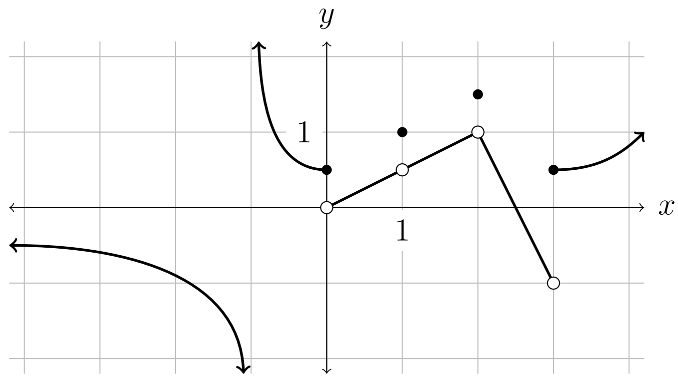
3. Use the intermediate value theorem to show that there are currently two points on the Earth's equator at exactly the same temperature.

4. Answer the following true / false statements with justification.

   (a) If a car travels 80 MPH between 2 PM and 4 PM, then its velocity is close to 40 MPH at 2 PM.

   (b) At some time since you were born your weight in pounds equaled your height in inches.

5. Determine where the following function is continuous. If it is not continuous at some points, determine, with proof (that is, using limit expressions), if it is continuous from the left and/or from the right at those points.



6. Answer the following true / false statements with justification.

   (a) A function can have two different limits at one x-value.

   (b) A rational function is continuous on its domain.

   (c) If a function f is not continuous at $x = a$, then either $\lim_{x \to a^-} [f(x)]$ or $\lim_{x \to a^+} [f(x)]$ does not exist.

7. Consider the function

$$f(x) = \begin{cases} (x-1)\sin\left(\frac{1}{x-1}\right) + 2 & \text{if } x \neq 1, \\ 2 & \text{if } x = 1. \end{cases}$$

   Is the function f continuous at $x = 1$? Give proof as to why or why not. (Hint: use the squeeze theorem)

8. **Bonus 1:** Why does an odd degree-polynomial always have at least one zero? (Hint: use the intermediate value theorem)

9. **Bonus 2:** Show, with proof, that $\sqrt{x^2 + 5}$ is not a polynomial. Recall that a polynomial is a finite sum $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ for some real numbers a_i.