ESP Math 179

Worksheet 2

Fall 2015

27 August 2015

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1. Answer the following questions with either "True" or "False." You may be asked to justify your reasoning.

   (a) If $\lim_{x\to 0}[f(x)] = 1$, then $\lim_{x\to 0}[f(x)-1] = 0$.

   (b) As long as $\lim_{x\to 0}[f(x)]$ exists, $\lim_{x\to 0}[2f(x)] \geqslant \lim_{x\to 0}[f(x)]$.

   (c) For any two functions f and g, $\lim_{x\to 5}[f(x)] + \lim_{x\to 5}[g(x)] = \lim_{x\to 5}[f(x)+g(x)]$.

   (d) If $\lim_{x\to -2^+}[f(x)] \neq \lim_{x\to -2^-}[f(x)]$, then $f(-2)$ is not defined.

   (e) If $\lim_{x\to 3}[h(x)]$ does not exist, then h does not have a tangent line at $x=3$.

   (f) If h does not have a tangent line at $x=3$, then $\lim_{x\to 3}[h(x)]$ does not exist.

For the next two questions, a function is considered to be "continuous" if you can draw it without lifting your pencil off the paper.

2. Make a grid to draw functions on.

   (a) Draw f such that $\lim_{x\to a}[f(x)]$ exists for all a, but f is not continuous at two points.

   (b) Draw g such that $\lim_{x\to 0}[g(x)-1]$ does not exist, and g is continuous everywhere except at one point.

   (c) Draw h such that $\lim_{x\to -1}[5h(x)] = \lim_{x\to 1}[h(x)+2]$ and $\lim_{x\to 0}[h(h(x))]$ does not exist, and h is defined at $x=0$.

3. Make a grid to draw functions on.

   (a) Draw $k(x)$ such that $\lim_{x\to 0^-}[k(x)] \neq \lim_{x\to 0^+}[k(x)] \neq k(0)$.

   (b) Draw $\ell(x)$ such that $\lim_{x\to 0^-}[\ell(x)]$ does not exist, but ℓ is continuous everywhere except at $x=0$.

   (c) Draw $m(x)$ such that $\lim_{x\to -2^-}[m(x)] = m\left(\lim_{x\to -2^+}[m(x)]\right)$.

4. (a) Give the equation of two different functions for which the limit does not exist at one point (the point may be different for each function).

   (b) Multiply the two functions you found in part (a) together.

      i. Does this function have points where the limit does not exist? If yes, are they the same points as in (a)?

      ii. Can you come up with two functions in part (a) such that their product does not have any points where the limit does not exist?

   (c) Repeat part (b) with addition instead of multiplication. What happens if the points where the limits do not exist for each function must be different?

5. Let $d(x)$ be the function that is 1 when x is a rational number and 0 when x is an irrational number.

   (a) Graph $d(x)$ from $x = -1$ to $x = 1$.

   (b) Does $\lim_{x\to a} [d(x)]$ exist for any $a \in [-1, 1]$?

   (c) Let $c(x) = xd(x)$ and graph $c(x)$.

   (d) Does $\lim_{x\to a} [c(x)]$ exist for any $a \in [-1, 1]$?