Assignment 7

Introduction to Linear Algebra

Material from Lectures 12 and 13
Due Thursday, March 2, 2023

12.1 (✠2.09) Let $A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ be a symmetric 4×4 matrix.

(a) Find which pairs of columns of A are orthogonal to each other.

(b) Give the nullspace of A as a span of the special solutions to $A\mathbf{x} = 0$.

(c) Show that the column space of A is orthogonal to the nullspace of A.

(d) Explain why for any symmetric matrix (not just the one given), its column space is orthogonal to its nullspace.

12.6 (✠2.10) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ -1 & -2 \\ 1 & 3 \end{bmatrix}$.

(a) What are the dimensions of $\text{col}(A)$ and $\text{col}(B)$? Only using dimensions, explain why $\text{col}(A) \neq \text{col}(B)^\perp$.

(b) Find a vector that is both in $\text{col}(A)$ and $\text{col}(B)$.
Hint: Row reduce the block matrix $\begin{bmatrix} A & B \end{bmatrix}$.

13.1 (✠2.11) This question is about repeated projections.

(a) Show that projecting twice onto a line is the same as projecting once.

(b) Show that projecting twice onto a subspace is the same as projecting once.

Hint: Use the projection matrices P from Equation (4) and Definition 13.6, and show that $P^2 = P$.

(c) Let $R_\theta \in \mathcal{M}_{2\times 2}$ be the rotation matrix from Example 12.7. For which $\theta \in [0, 2\pi)$ is R_θ a projection matrix? Justify your answer.

13.5 (✠2.12) Let $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ 2 & 1 & 1 & -2 \\ -1 & 0 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \\ 1 & -1 \\ 0 & -1 \\ 0 & 2 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$.

(a) Compute the projection of $\mathbf{v}$ onto $\text{col}(A)$ and $\text{col}(A)^\perp$. What is the angle between the two projections?

(b) Compute the projection of $\text{col}(B)$ onto $\text{col}(A)$.