Assignment 6
Introduction to Linear Algebra

Material from Lectures 10 and 11
Due Thursday, February 16, 2023

10.1 (✠2.03) Find all sets of size 3 from the vectors below that are linearly independent:

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}.$$

10.5 (✠2.04) This question is about expressing vectors in different bases.

(a) Express the vector $\begin{bmatrix} 3 \\ -2 \\ -8 \end{bmatrix}$ in the basis $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

(b) There are two bases $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ and $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ of a vector space V, with the following relations:

$$\mathbf{b}_1 = \mathbf{a}_1 + \mathbf{a}_2, \qquad \mathbf{b}_2 = \mathbf{a}_2 + \mathbf{a}_3, \qquad \mathbf{b}_3 = \mathbf{a}_1 + \mathbf{a}_3.$$

If you know that $\mathbf{v} = 3\mathbf{a}_1 - 2\mathbf{a}_2 - 8\mathbf{a}_3$, express $\mathbf{v}$ as a linear combination of $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$.

11.1 (✠2.07) Consider the plane $P = \{(x, y, z) \in \mathbf{R}^3 \ : \ 2x - 4y - 5z = 0\}$, which is a subspace of $\mathbf{R}^3$.

(a) Find a vector $\mathbf{n}$ normal to the plane P. That is, find $\mathbf{n} \in \mathbf{R}^3$ so that $\mathbf{n} \bullet \mathbf{v} = 0$, for $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ a solution to $2x - 4y - 5z = 0$.

(b) Considering the vector $\mathbf{n}$ as a 1×3 matrix A, the nullspace of A is precisely all points in the plane P. Find this nullspace, and express it as a span.

(c) What is a basis for P?

11.2 (✠2.08) For $a, b, c \in \mathbf{R}$, consider the matrix

$$A = \begin{bmatrix} 0 & 1 & a & 0 & a & 0 \\ 0 & 0 & 1 & b & 0 & b \\ 0 & 0 & 0 & 1 & c & c \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for the column space, nullspace, row space, and left nullspace of A.

(b) Do the dimensions of the four fundamental spaces change if all of a, b, c are zero?