Assignment 4

Introduction to Linear Algebra

Material from Lectures 6 and 7
Due Thursday, February 2, 2023

6.2 (✠2.02) Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be three different vectors in a vector space V. Consider the three spans $S_1 = \operatorname{span}(\{\mathbf{u} - \mathbf{v}\})$, $S_2 = \operatorname{span}(\{\mathbf{u}, \mathbf{v}, \mathbf{w}\})$ and $S_3 = \operatorname{span}(\{\mathbf{u} + \mathbf{v}, \mathbf{v} + \mathbf{w}\})$.

(a) Show that $S_1 \subseteq S_2$.

(b) Show that $S_3 \subseteq S_2$.

(c) For $V = \mathbf{R}^3$, given an example of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ for which $S_2 = S_3$.

(d) For $V = \mathbf{R}^3$, given an example of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ for which all of S_1, S_2, S_3 are different.

6.3 (✠2.01) Consider the set X of all functions $f\colon \mathbf{R} \to \mathbf{R}$.

(a) If addition on X is defined as $(f + g)(x) = f(x) + g(x)$ and multiplication is defined as $(cf)(x) = f(cx)$, show that X can not be a vector space.

(b) If multiplication is instead defined as $(cf)(x) = cf(x)$, and addition is instead defined as $(f + g)(x) = f(g(x))$ show that X still can not be a vector space.

Hint: Show X is not a vector space with examples!

7.1 (✠1.12) Consider the matrix $A = \begin{bmatrix} 2 & 9 & 1 & 0 & 0 & 9 \\ 0 & -3 & 0 & 1 & 0 & -3 \\ 8 & -6 & 0 & 0 & 1 & 1 \end{bmatrix}$.

(a) Construct the column space of A as a span of three vectors.

(b) Construct the nullspace of A as a span of vectors.

7.6 (✠1.13) Let X be a set of 2×2 matrices defined in the following way:

- $\left[\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right] \in X$
- if $M \in X$, then $MM^T \in X$
- if $M, N \in X$, then $aM + bN \in X$, for any $a, b \in \mathbf{R}$

Using scalar multiplication and matrix addition as in $\mathcal{M}_{2\times 2}$, show that X is a vector subspace of $\mathcal{M}_{2\times 2}$.
Hint: Using the given facts, try to construct the four special matrices that generate $\mathcal{M}_{2\times 2}$.