Assignment 3

Introduction to Linear Algebra

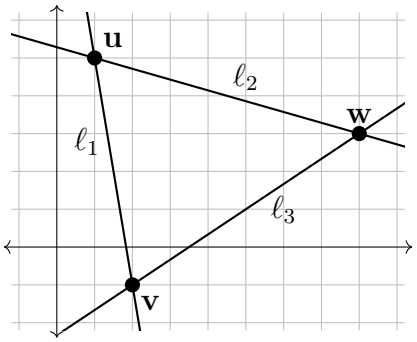
Material from Lectures 4 and 5
Due Thursday, January 26, 2023

4.1 (✠1.08) Consider the matrix equation $A\mathbf{x} = \mathbf{b}$, given by $\begin{bmatrix} 3 & -1 & 2 \\ 9 & -3 & 2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ -3 \end{bmatrix}$. Use Gaussian elimination on the augmented matrix $[A \mid \mathbf{b}]$ to solve for x, y, z.

4.4 (✠1.09) Using Gauss–Jordan elimination, find the inverse matrix of $A = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 0 & -4 \\ 2 & 2 & 2 \end{bmatrix}$.

5.2 (✠1.10) Decompose $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ into $PA = LDU$ factorization.

5.6 (✠1.06, 1.11) Consider three points $\mathbf{u} = (1, 5), \mathbf{v} = (2, -1), \mathbf{w} = (8, 3)$ in $\mathbf{R}^2$. Let ℓ_1 be the line through $\mathbf{u}$ and $\mathbf{v}$, ℓ_2 be the line through $\mathbf{u}, \mathbf{w}$, and ℓ_3 be the line through $\mathbf{v}, \mathbf{w}$, as in the diagram below.



(a) Give the matrix equation for which the lines in the diagram above are the row picture.

(b) Without solving this matrix equation, explain why the the equation has no solutions.

(c) Now suppose that $\mathbf{u} = (5, 1)$. Give the new matrix equation (the lines ℓ_1, ℓ_2, ℓ_3 are constructed in the same way), and again, without solving it, explain why it has infinitely many solutions.