Homework 2

Introduction to Linear Algebra

Material from Lectures 2 and 3
Due Thursday, January 19, 2023

2.1 (✠1.03) Let A, B, C, D be $n \times n$ matrices that are invertible. Find the inverses of the following block matrices.

(a) $\begin{bmatrix} I & 0 \\ 0 & D \end{bmatrix}$ (b) $\begin{bmatrix} I & B \\ 0 & D \end{bmatrix}$ (c) $\begin{bmatrix} A & 0 \\ I & D \end{bmatrix}$

2.5 *Removed*

2.6 (✠1.05) For each part of this question, construct a Python function with the given name.

(a) Make a function `ones_counter(matrix)` that takes in a matrix, in the form of a `numpy` array, and returns the number of entries that are `1`.

(b) Make a function `thick_diagonal(rownum, colnum)` that takes in two positive integers and returns a matrix, in the form of a `numpy` array, having `rownum` rows and `colnum` columns, and zero everywhere except on the diagonal and just above and just below it. For example, `thick_diagonal(5, 10)` should return the following matrix:

```
array([[1, 1, 0, 0, 0, 0, 0, 0, 0, 0],
       [1, 1, 1, 0, 0, 0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 0, 0, 0, 0, 0],
       [0, 0, 0, 1, 1, 1, 0, 0, 0, 0]])
```

3.3 (✠1.07) Let $a, b, c \in \mathbf{R}$ be nonzero numbers, and consider the matrix $A = \begin{bmatrix} a & b & c \\ a & 2b & 3c \\ a & 3b & 6c \end{bmatrix}$.

(a) Give the elementary matrices which, when they are multiplied on the left of A, leave A with zeros below the diagonal (not above).

(b) Let E be the product of the elementary matrices you computed in the first part of this question, and suppose that you began with an equation $A\mathbf{x} = \mathbf{b}$. If $E\mathbf{b} = \begin{bmatrix} 47 \\ 4 \\ 7 \end{bmatrix}$, what is $\mathbf{b}$? What are a, b, c?