Assignment 12

Introduction to Linear Algebra

Material from Lectures 22 and 23

Due Thursday, April 6, 2023

22.2 (✠3.08) Let A, B, C be any 3×3 matrices, with C diagonalizable.

(a) Show that $\text{trace}(AB) = \text{trace}(BA)$.

(b) Use that above to show that $\text{trace}(C)$ is the sum of the three eigenvalues of C. *Hint: Split up the diagonalization of C into two matrices.*

(c) Suppose that the eigenvalues of C are $1, \frac{1}{2}, \frac{1}{3}$. Show why the limit $\lim_{n\to\infty} C^n$ exists, and why it has rank 1.

22.3 (✠3.09) Decompose both matrices below in their $X\Lambda X^{-1}$-decomposition, where Λ is a diagonal matrix with the eigenvalues, and X is the matrix with columns as eigenvectors.

$$A = \begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix} \qquad\qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

23.1 (✠3.10) The three vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ are linearly independent.

(a) Construct a matrix A with eigensystem $\{(\mathbf{u}, 2), (\mathbf{v}, -1), (\mathbf{w}, 3)\}$.

(b) Give examples of two matrices B, C that are similar to A.

23.6 (✠3.08, 3.12) Consider the symmetric matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix}$.

(a) Find the trace and determinant of A. Do not use a calculator, show your work.

(b) Diagonalize A as $Q\Lambda Q^T$.

(c) Express A as a sum of rank one matrices using the part above.